

Controllability and invariance of monotone systems for robust ventilation automation in buildings

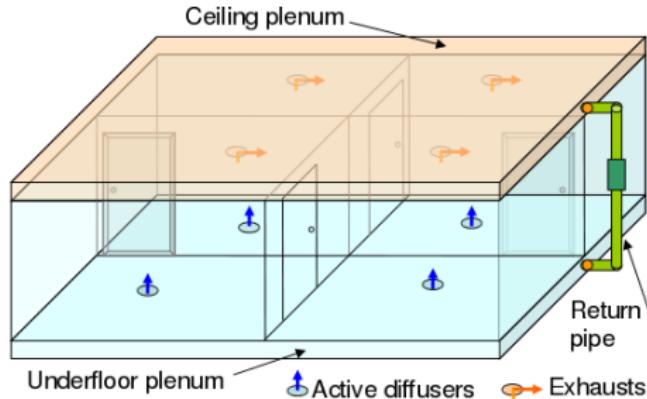
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UnderFloor Air Distribution



- Underfloor air cooled down
- Sent into the rooms by fans
- Air excess pushed through the ceiling exhausts
- Returned to the underfloor
- Disturbances: heat sources; opening of doors

Model

Temperature variations in room i :

- energy conservation;
- mass conservation.

Model

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$$\begin{aligned}\frac{dT_i}{dt} = & \sum_j a_{i,j}(T_j - T_i) && \text{Conduction through walls} \\ & + b_i u_i (T_u - T_i) && \text{Controlled fan air flow } u_i \\ & + \sum_j \delta_{d_{ij}} c_{i,j} * h(T_j - T_i) && \text{Open doors (flow hot} \rightarrow \text{cold)} \\ & + \delta_{s_i} d_i (T_{s_i}^4 - T_i^4) && \text{Radiation from heat sources}\end{aligned}$$

Model

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$$\begin{aligned}\frac{dT_i}{dt} = & \sum_j a_{i,j}(T_j - T_i) && \text{Conduction through walls} \\ & + b_i u_i (T_u - T_i) && \text{Controlled fan air flow } u_i \\ & + \sum_j \delta_{d_{ij}} c_{i,j} * h(T_j - T_i) && \text{Open doors (flow hot} \rightarrow \text{cold)} \\ & + \delta_s d_i (T_{s_i}^4 - T_i^4) && \text{Radiation from heat sources}\end{aligned}$$

- $a, b, c, d > 0$;
- δ_s, δ_d : discrete state of the disturbances (heat sources and doors);
- $\begin{cases} h(x \leq 0) = 0 \\ h(x > 0) = x^{3/2} \end{cases}$: door heat transfer only in the colder room.

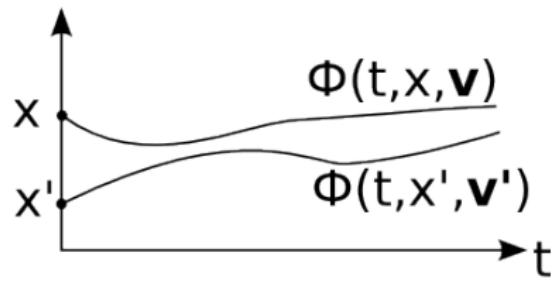
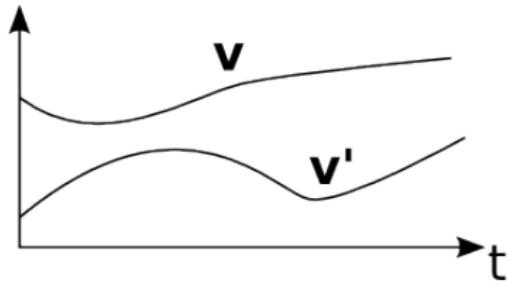
Monotonicity

Generic system $\dot{x} = f(x, v)$ with trajectories $\Phi(t, x, v)$.

Definition (Monotonicity)

The system Φ is monotone if its trajectories preserve some partial orders:

$$v \succeq_v v', x \succeq_x x' \Rightarrow \forall t \geq 0, \Phi(t, x, v) \succeq_x \Phi(t, x', v')$$



Monotonicity

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Definition (Partial order)

$$x \succeq_x x' \Leftrightarrow \forall i, (-1)^{\varepsilon_i}(x_i - x'_i) \geq 0, \quad \text{with } \varepsilon_i \in \{0, 1\}$$

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Proposition (Angeli and Sontag, 2003)

The system defined by $\dot{x} = f(x, v)$ is monotone if and only if,

$$\forall x \in \mathbb{R}^n, \forall v \in \mathbb{R}^m, \begin{cases} (-1)^{\varepsilon_i + \varepsilon_j} \frac{\partial f_i}{\partial x_j}(x, v) \geq 0, & \forall i, \forall j \neq i, \\ (-1)^{\varepsilon_i + \gamma_k} \frac{\partial f_i}{\partial v_k}(x, v) \geq 0, & \forall i, \forall k. \end{cases}$$

Where $\varepsilon \in \{0, 1\}^n$ and $\gamma \in \{0, 1\}^m$ define the partial orders for x and v .

Monotonicity

Our model: $\dot{T} = f(T, u, w, \delta)$

- T : state (temperature);
- u : controlled input (fan air flow);
- w : exogenous input (other temperatures);
- δ : discrete disturbance embedded in a continuous space.

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$$\begin{aligned} T \succeq_T T' &\Leftrightarrow \forall i, T_i \geq T'_i \\ \mathbf{u} \succeq_u \mathbf{u}' &\Leftrightarrow \forall t \geq 0, \forall k, \mathbf{u}_k(t) \leq \mathbf{u}'_k(t) \\ \mathbf{w} \succeq_w \mathbf{w}' &\Leftrightarrow \forall t \geq 0, \forall k, \mathbf{w}_k(t) \geq \mathbf{w}'_k(t) \\ \boldsymbol{\delta} \succeq_{\boldsymbol{\delta}} \boldsymbol{\delta}' &\Leftrightarrow \forall t \geq 0, \forall k, \delta_k(t) \geq \delta'_k(t) \end{aligned}$$

$$\Phi(t, T, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}) \succeq_T \Phi(t, T', \mathbf{u}', \mathbf{w}', \boldsymbol{\delta}')$$

Robust Invariance

Definition (Robust Invariance)

The system is *Robust Invariant* in an interval $[\underline{T}_r, \overline{T}_r]$ if,

$$\begin{aligned} \forall T_0 \in [\underline{T}_r, \overline{T}_r], \quad & \forall \mathbf{w} \in [\underline{\mathbf{w}}, \overline{\mathbf{w}}], \quad \forall \delta \in [\underline{\delta}, \overline{\delta}], \quad \forall \mathbf{u} \in [\underline{\mathbf{u}}, \overline{\mathbf{u}}], \\ \forall t \geq 0, \quad & \Phi(t, T_0, \mathbf{u}, \mathbf{w}, \delta) \in [\underline{T}_r, \overline{T}_r]. \end{aligned}$$

Proposition

The minimal Robust Invariant interval $[\underline{T}_r, \overline{T}_r]$ is given by

$$\begin{cases} f(\overline{T}_r, \underline{\mathbf{u}}, \overline{\mathbf{w}}, \overline{\delta}) = 0 \\ f(\underline{T}_r, \overline{\mathbf{u}}, \underline{\mathbf{w}}, \underline{\delta}) = 0 \end{cases}$$

Robust Controlled Invariance

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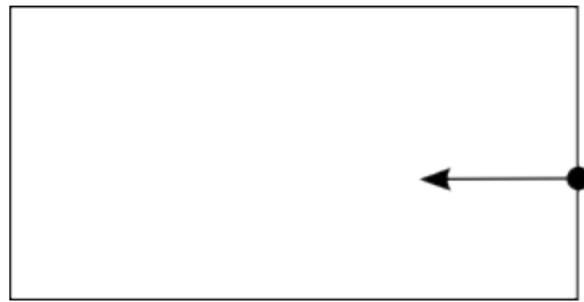
$$\begin{aligned} & \forall T_0 \in [\underline{T}, \bar{T}], \quad \forall \mathbf{w} \in [\underline{\mathbf{w}}, \bar{\mathbf{w}}], \quad \forall \boldsymbol{\delta} \in [\underline{\boldsymbol{\delta}}, \bar{\boldsymbol{\delta}}], \\ & \exists \mathbf{u} \in [\underline{u}, \bar{u}] \mid \forall t \geq 0, \quad \Phi(t, T_0, \mathbf{u}, \mathbf{w}, \boldsymbol{\delta}) \in [\underline{T}, \bar{T}]. \end{aligned}$$

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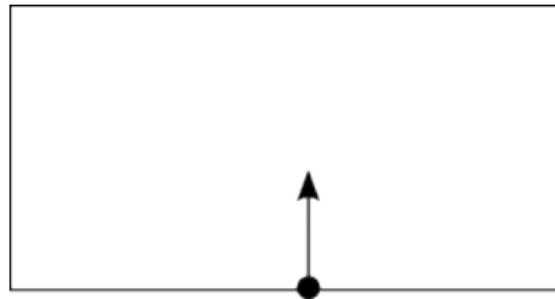


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The system is Robust Controlled Invariant in $[\underline{T}, \overline{T}]$ if and only if

$$\forall i, \quad \begin{cases} f_i(\overline{T}, \overline{u_i}, \overline{w}, \overline{\delta}) \leq 0 \\ f_i(\underline{T}, \underline{u_i}, \underline{w}, \underline{\delta}) \geq 0 \end{cases}$$

Robust Controlled Invariance

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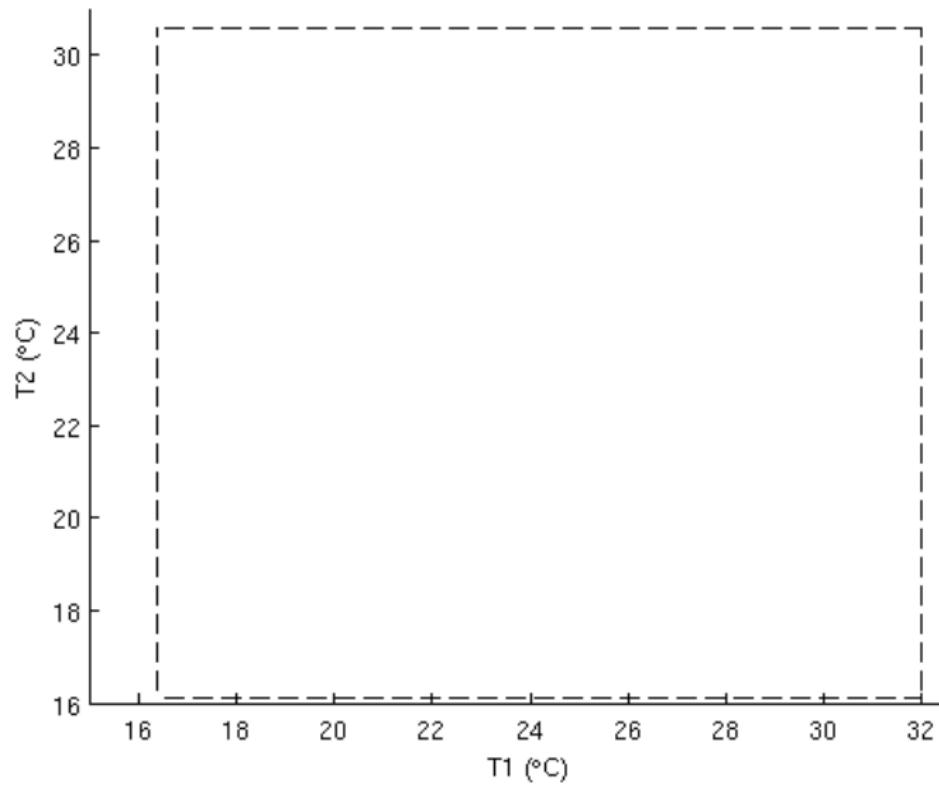
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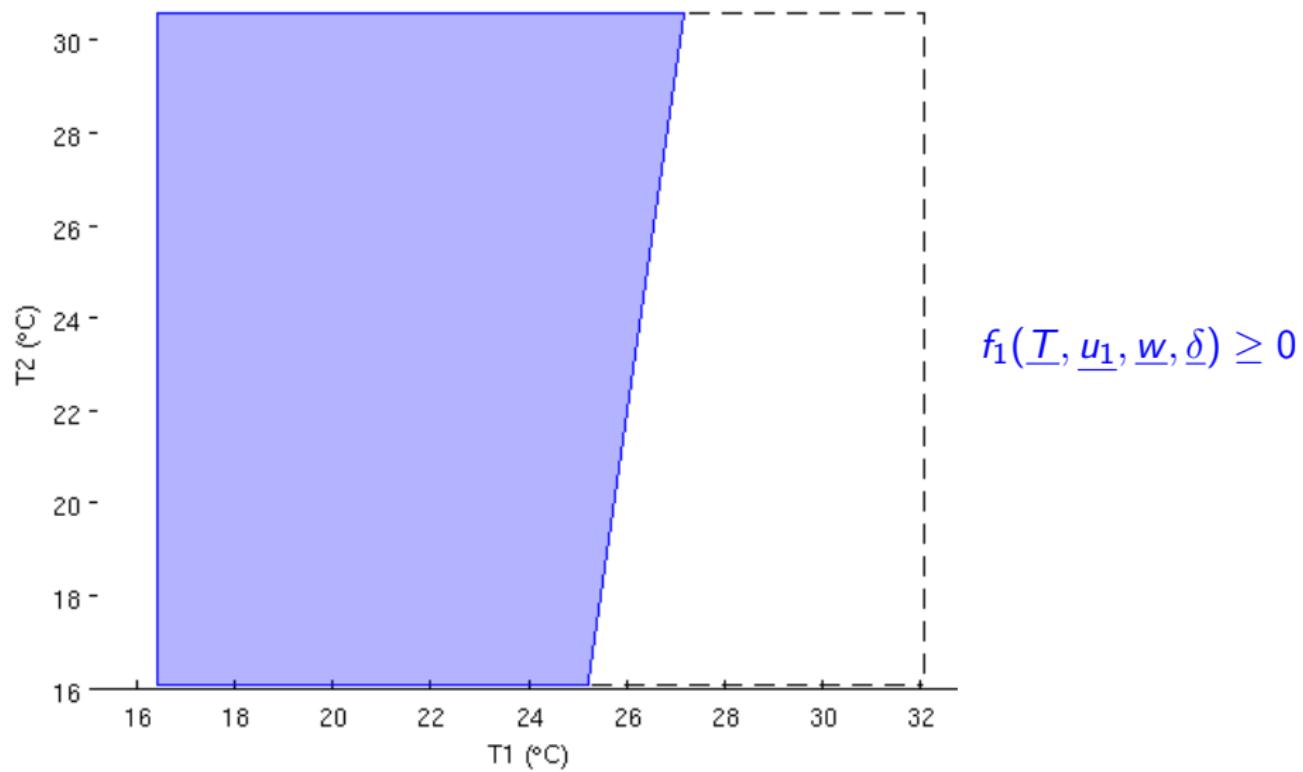
Definition (Decentralized Bang-Bang Controller)

$$\forall i, \quad \begin{cases} T_i \geq \overline{T}_i \Rightarrow u_i = \overline{u_i} \\ T_i \leq \underline{T}_i \Rightarrow u_i = \underline{u_i} \end{cases}$$

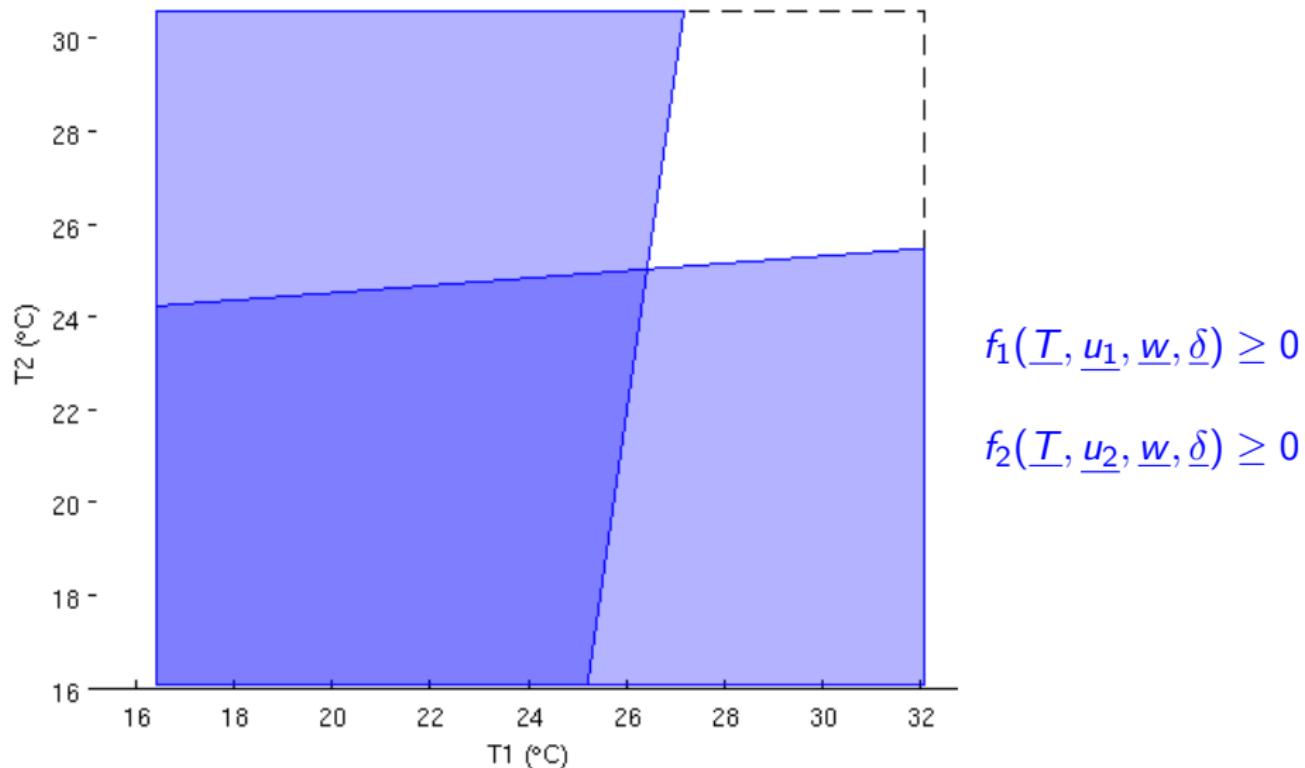
Controllable Spaces (2-room example)



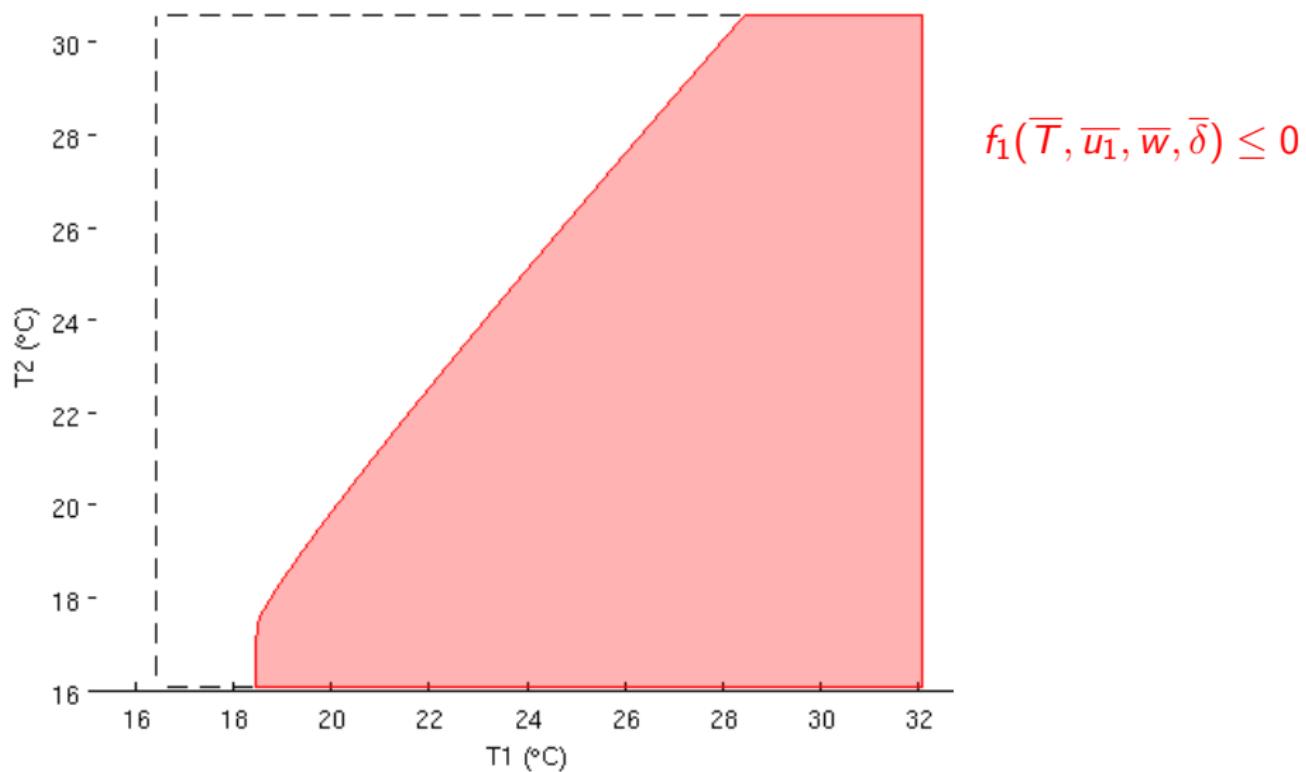
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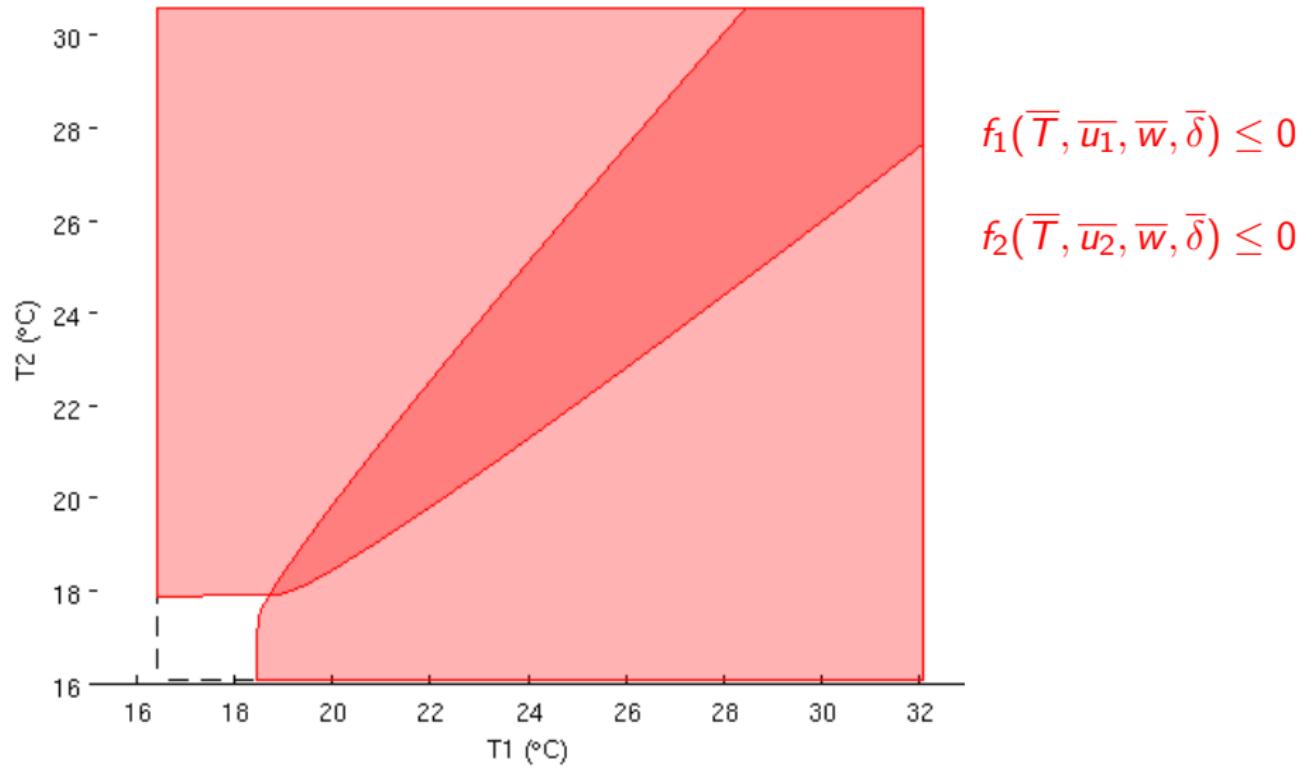
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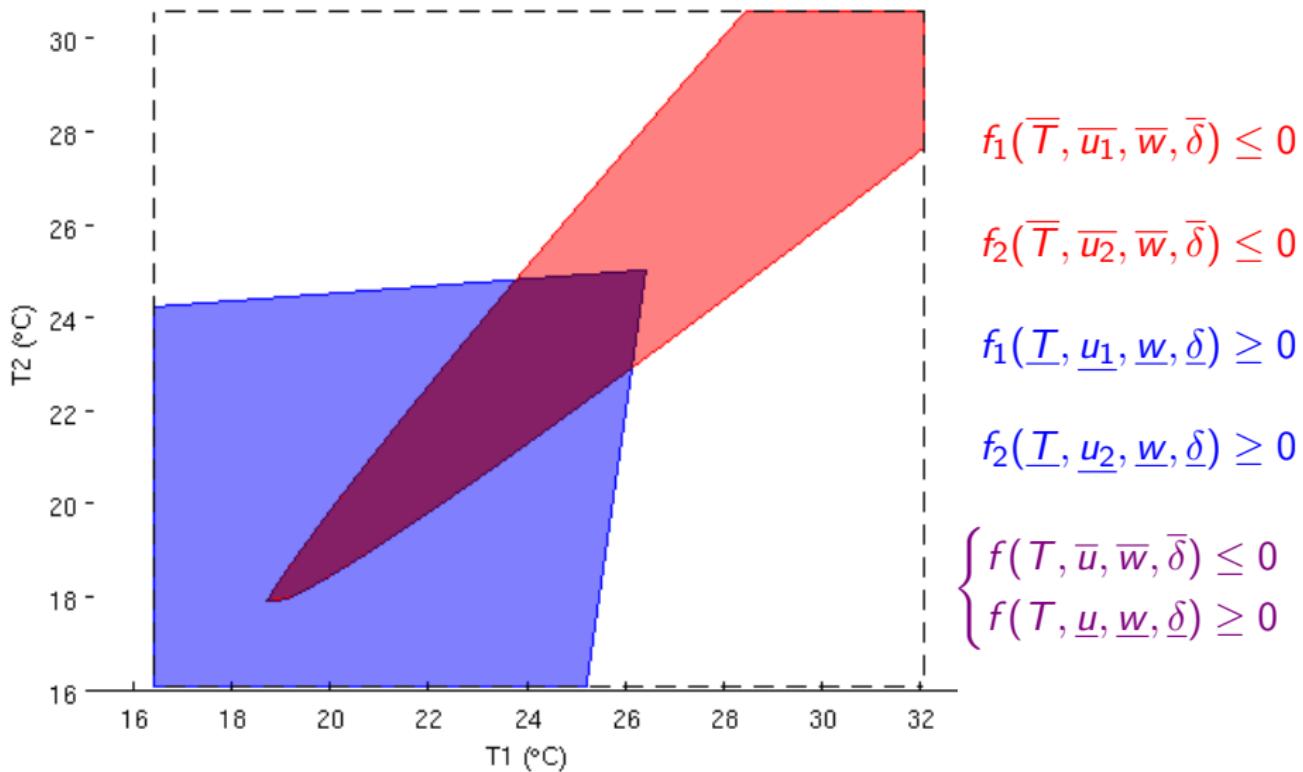
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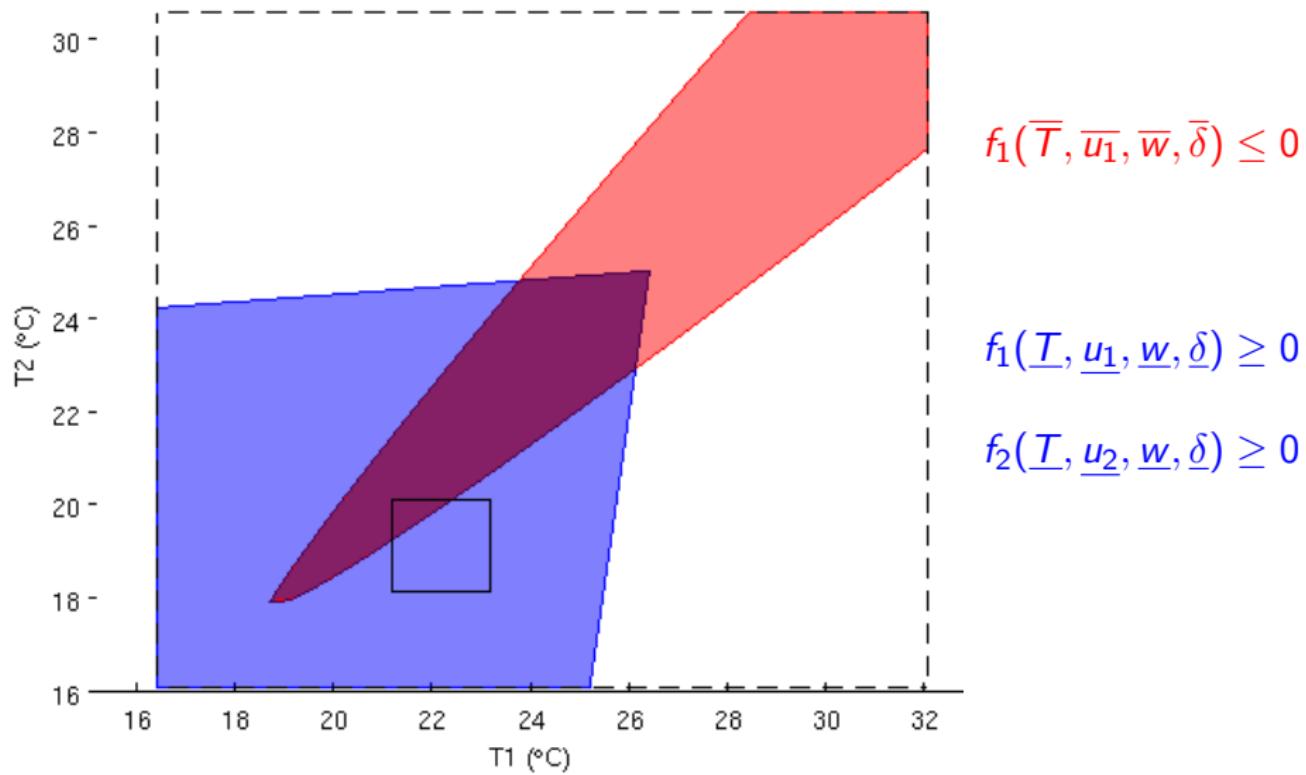
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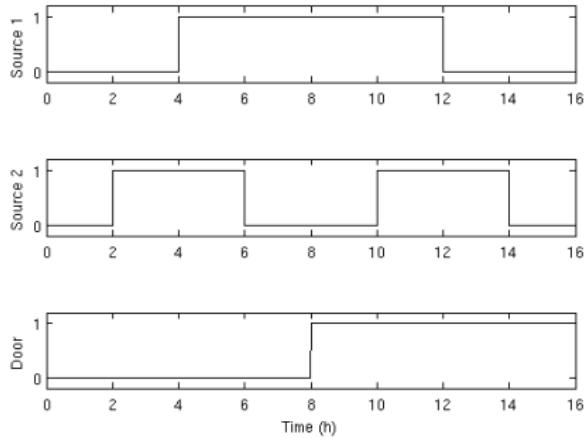


Control simulation

3 discrete disturbances:

- heat source in room 1
- heat source in room 2
- door

8 possible combinations

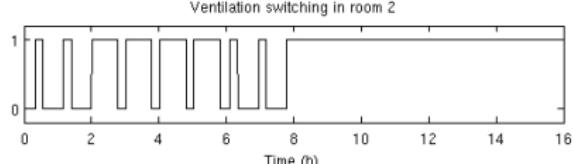
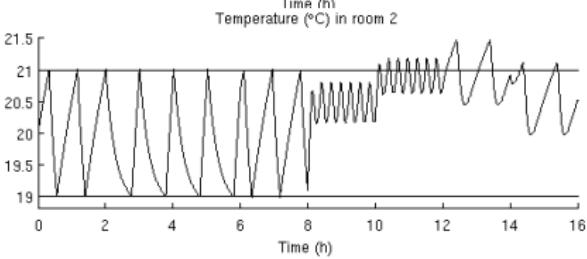
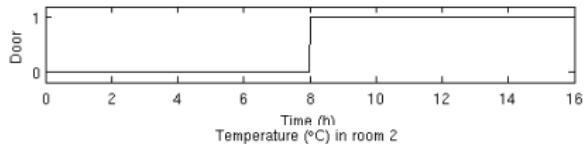
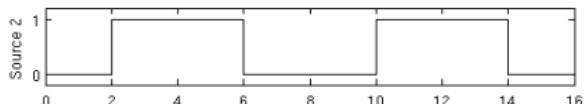
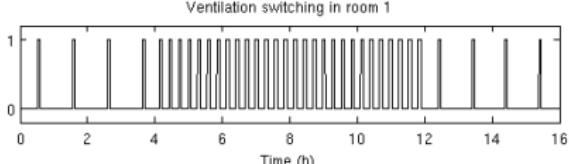
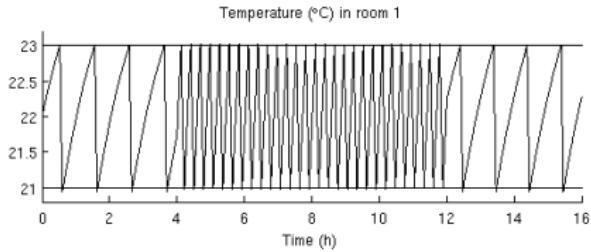


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Conclusion and perspectives

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- for a class of monotone systems,
- with decentralized control,
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Perspectives

- control strategy for energy reduction;
- stabilization;
- application to an experimental building.