



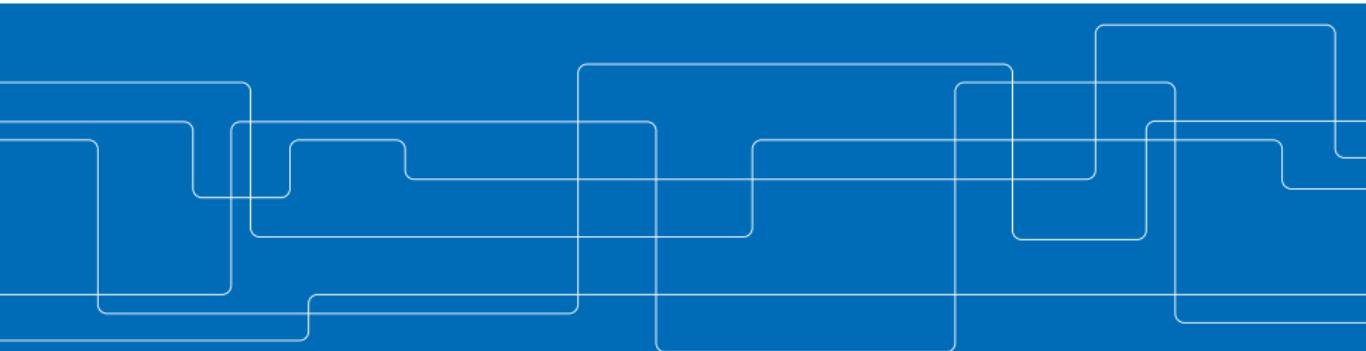
Knut och Alice
Wallenbergs
Stiftelse

Compositional abstraction refinement for control synthesis under lasso-shaped specifications

Pierre-Jean Meyer Dimos V. Dimarogonas

KTH, Royal Institute of Technology

May 24th 2017





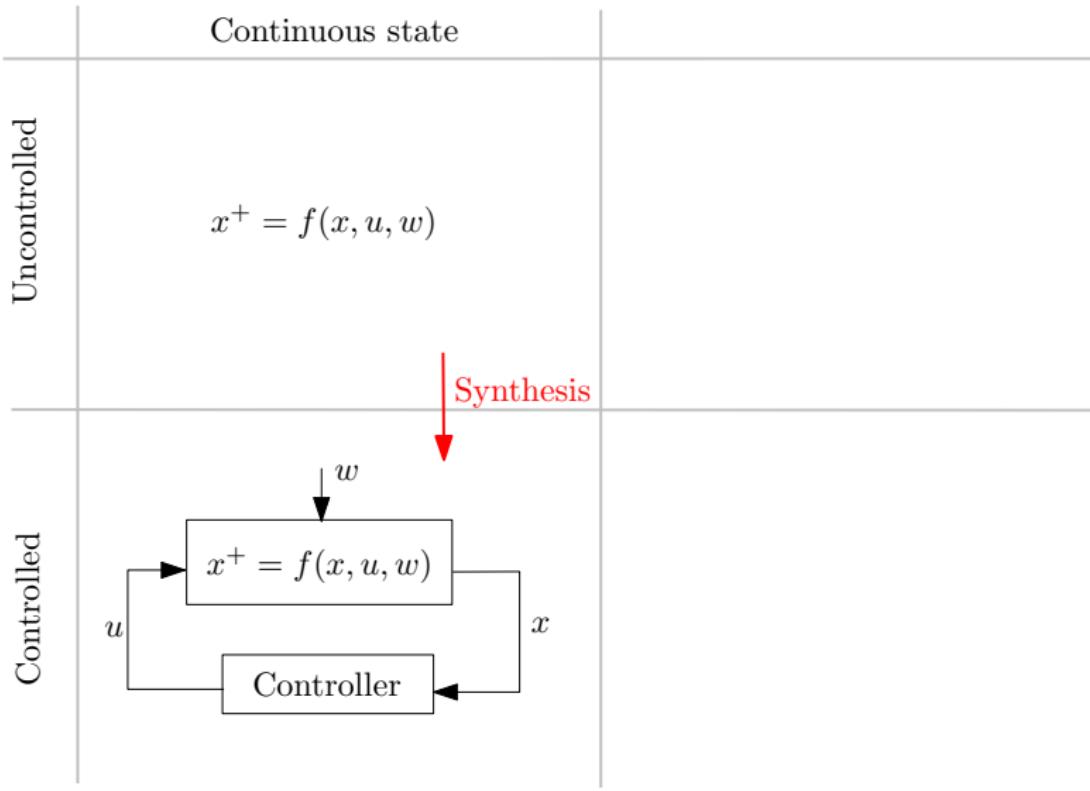
Outline

Context and intuition

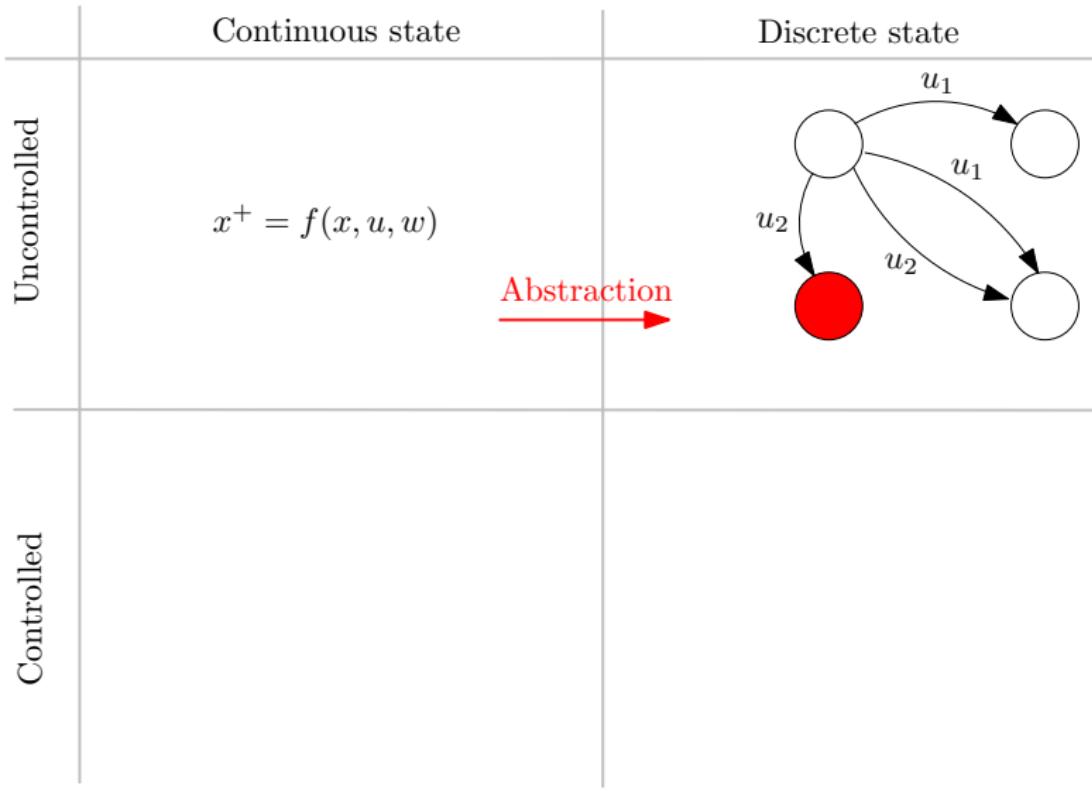
Abstraction refinement algorithm

Compositional approach

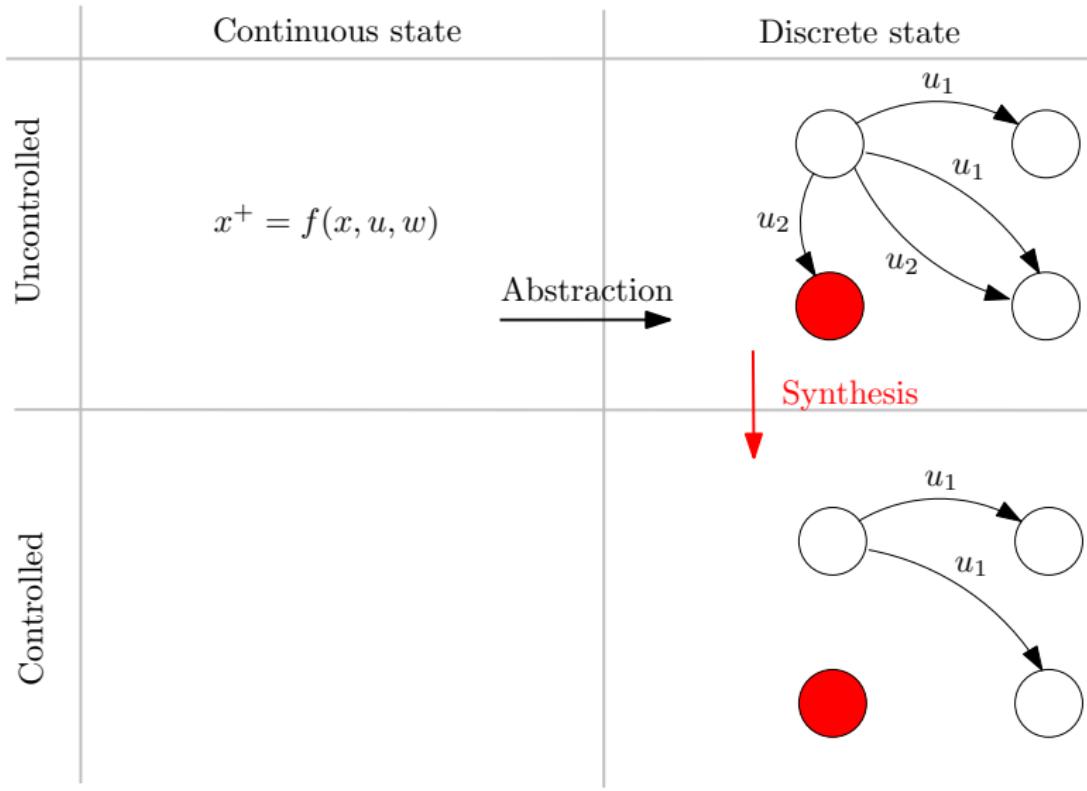
Abstraction-based synthesis



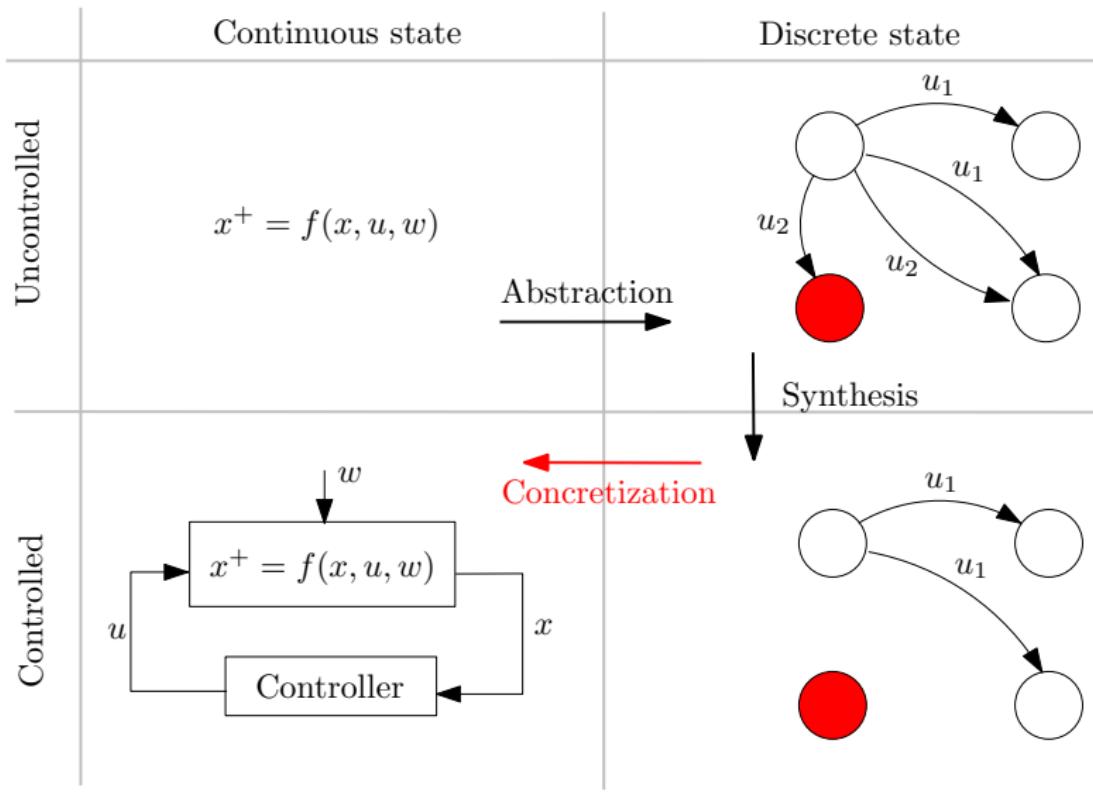
Abstraction-based synthesis



Abstraction-based synthesis



Abstraction-based synthesis



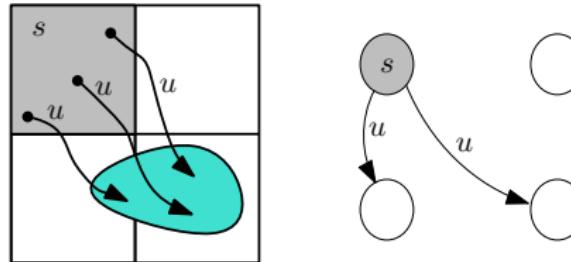
Abstraction procedure

$$x^+ = f(x, u, w)$$

- ▶ Define the **reachable set** for any disturbance $w \in W$:

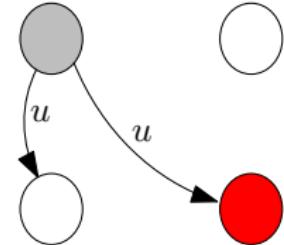
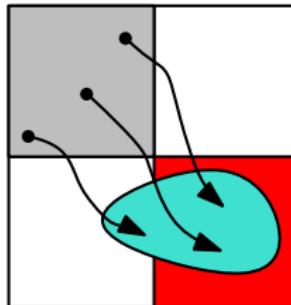
$$RS(X, U) = \{f(x, u, w) \mid x \in X, u \in U, w \in W\}$$

- ▶ Partition of the state space
- ▶ For each partition cell s and control u : compute the **reachable set** $RS(s, \{u\})$
- ▶ Obtain a **non-deterministic** transition system: each pair (s, u) may have several successors

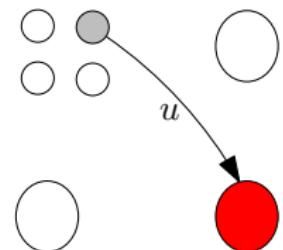
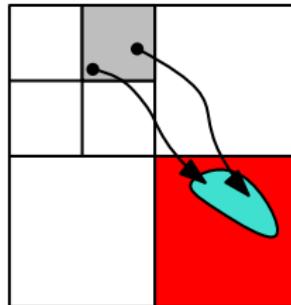


Abstraction refinement

No guarantee of reaching the red cell



Refine the abstraction by
splitting the partition





Outline

Context and intuition

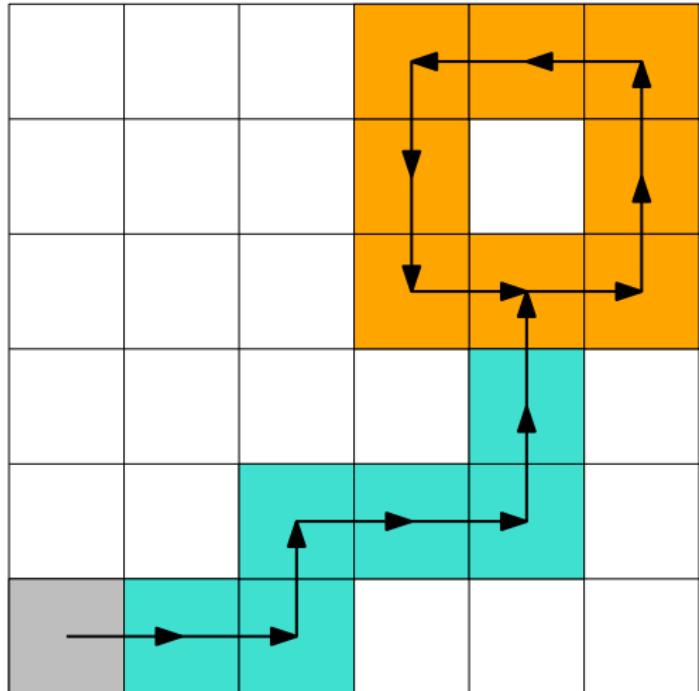
Abstraction refinement algorithm

Compositional approach

Specifications

Follow a **lasso-shaped** sequence in the state partition

- ▶ **prefix**: finite path from the initial cell
- ▶ **suffix**: finite path looping on itself

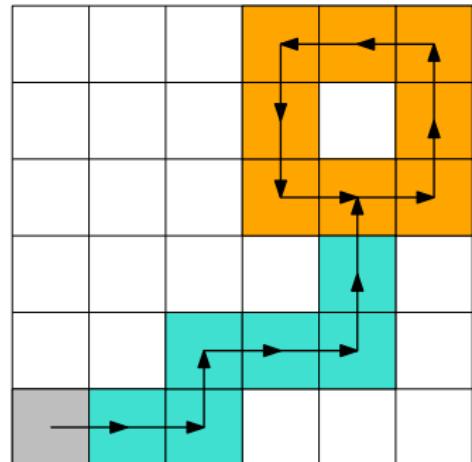


Valid sets

Definition (Valid set)

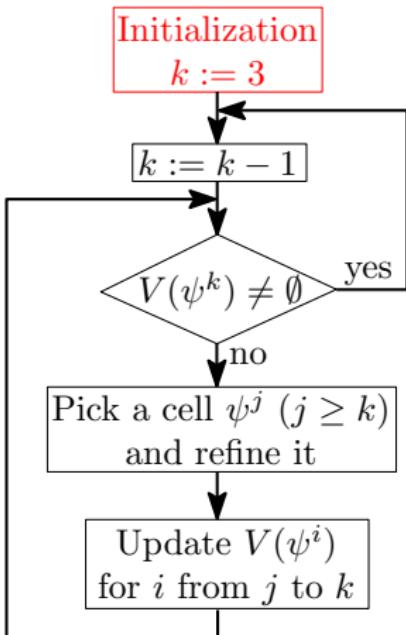
Elements of the refined partition X_a that can be controlled to follow the desired sequence of cells

Lasso sequence: $\psi = \psi^0\psi^1\psi^2\dots$

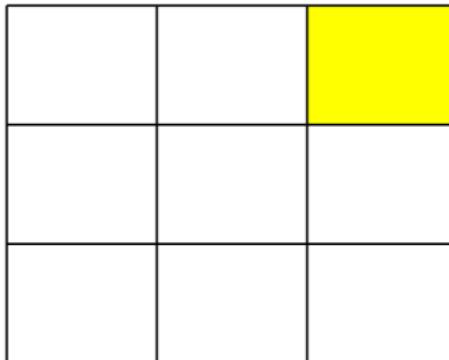
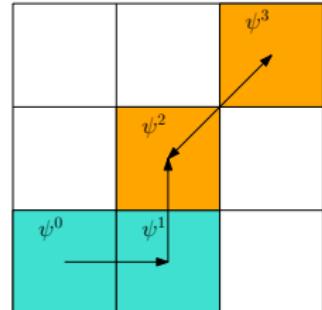


$$V(\psi^k) = \{s \in X_a | s \subseteq \psi^k, \exists u \in U_a, RS(s, \{u\}) \subseteq V(\psi^{k+1})\}$$

Algorithm

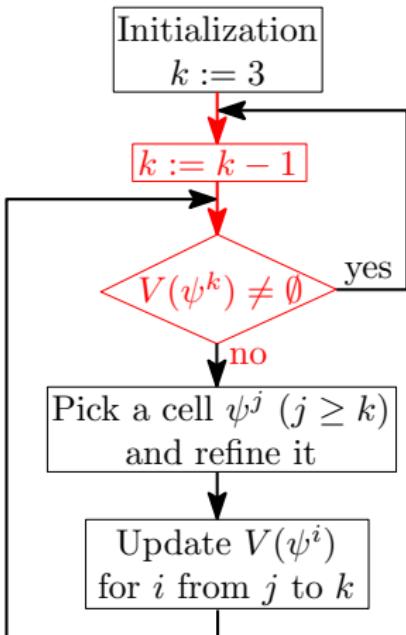


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

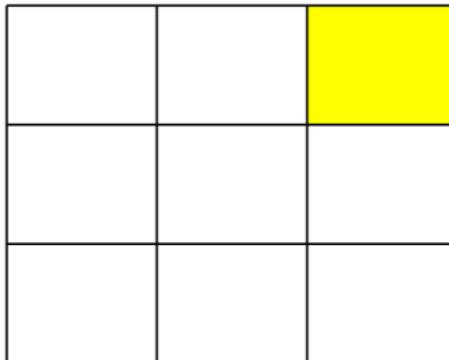
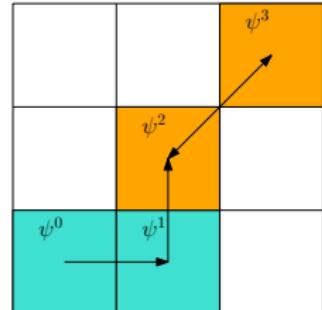


Last step of the suffix: ψ^3
Initial valid set: $V(\psi^3) = \{\psi^3\}$

Algorithm

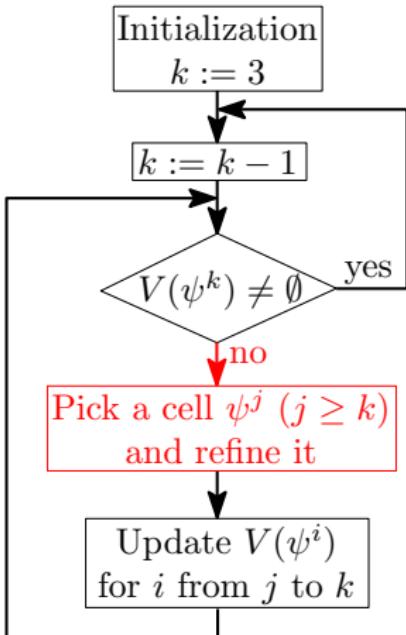


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

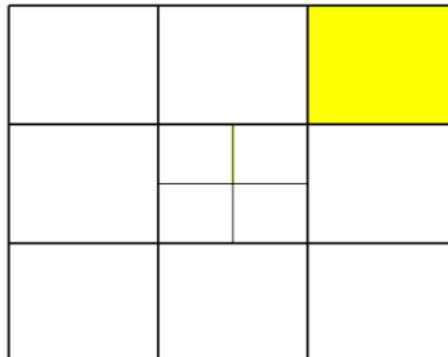
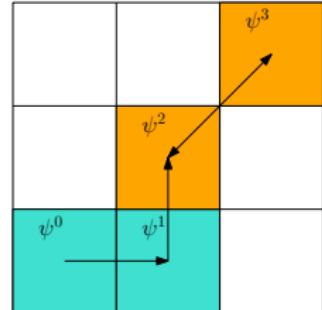


Transition $\psi^2 \rightarrow \psi^3$
Empty valid set: $V(\psi^2) = \emptyset$

Algorithm

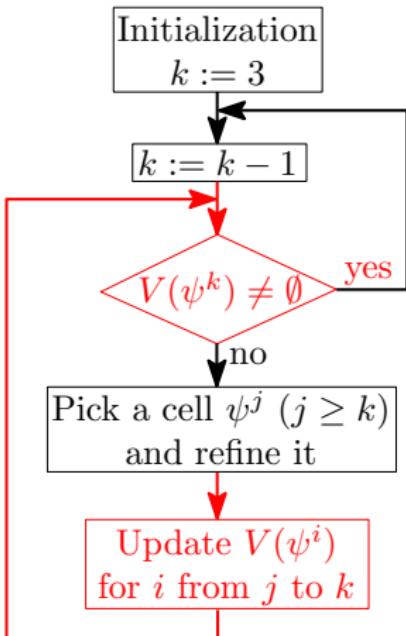


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

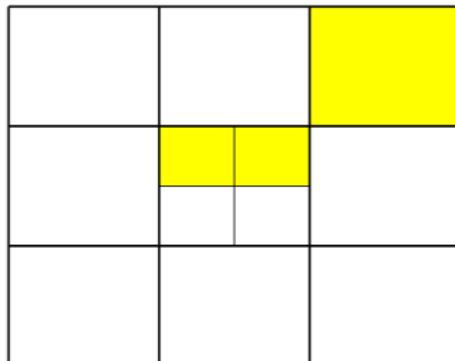
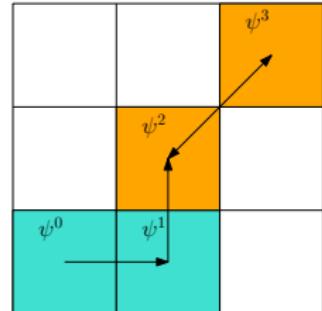


Transition $\psi^2 \rightarrow \psi^3$
Refine ψ^2

Algorithm

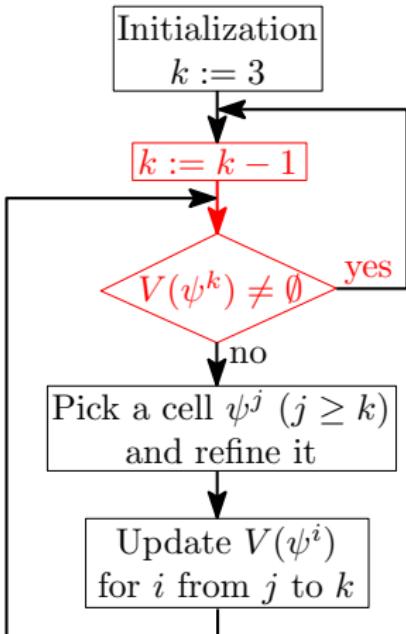


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

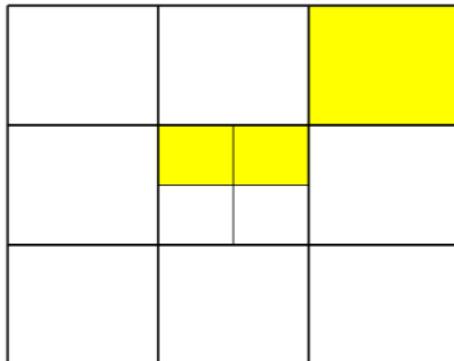
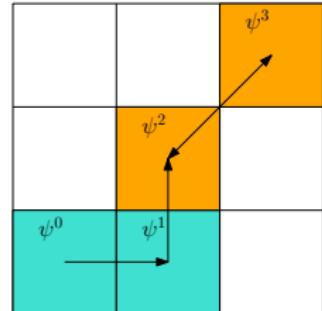


Transition $\psi^2 \rightarrow \psi^3$
 New valid set for ψ^2 : $V(\psi^2) \neq \emptyset$

Algorithm

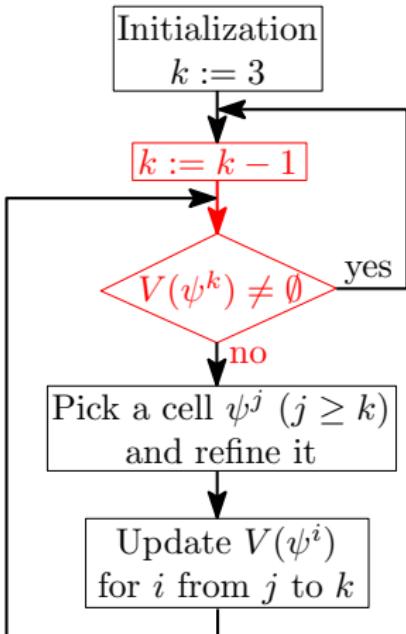


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

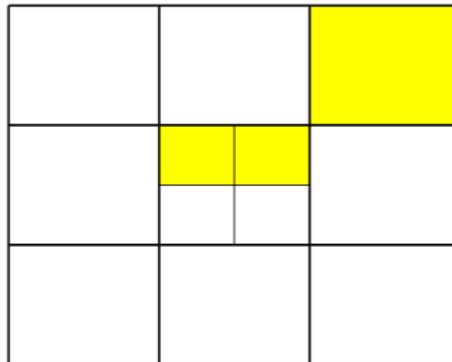
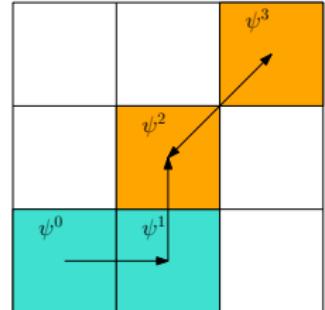


Transition $\psi^3 \rightarrow \psi^2$ (suffix loop)
 Unchanged valid set $V(\psi^3)$
 Loop on suffix finished

Algorithm

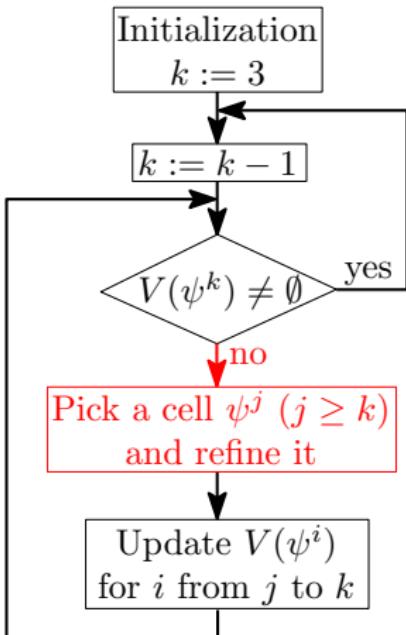


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

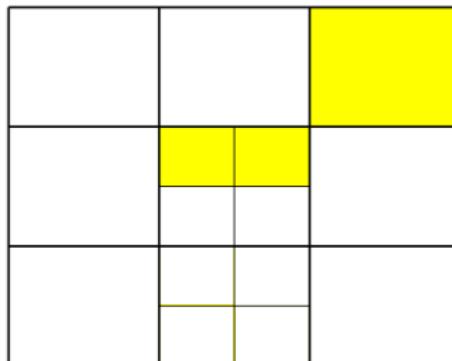
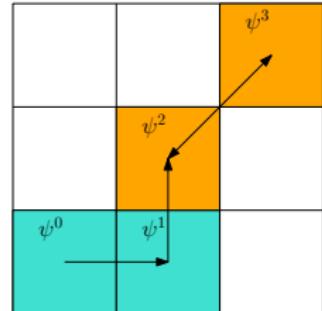


Transition $\psi^1 \rightarrow \psi^2$
Empty valid set: $V(\psi^1) = \emptyset$

Algorithm

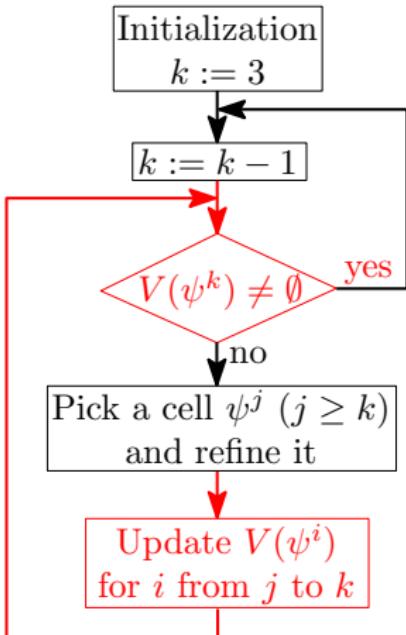


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

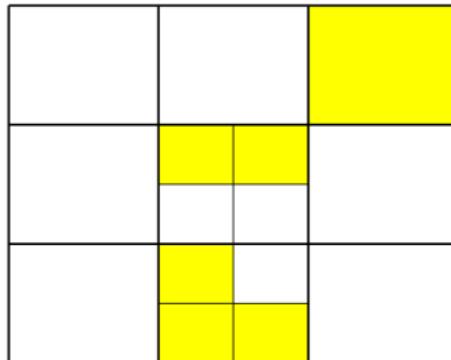
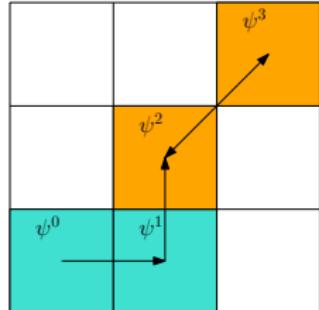


Transition $\psi^1 \rightarrow \psi^2$
Refine ψ^1

Algorithm

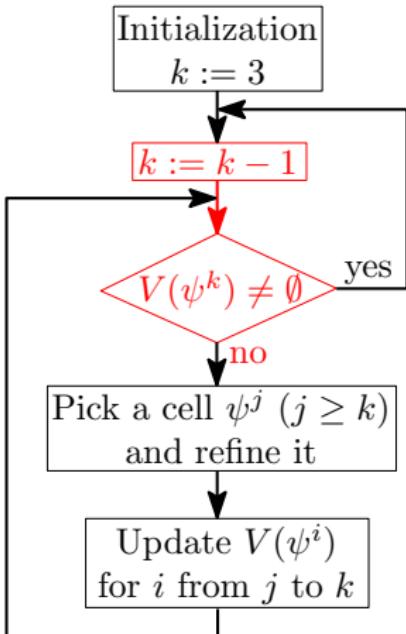


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

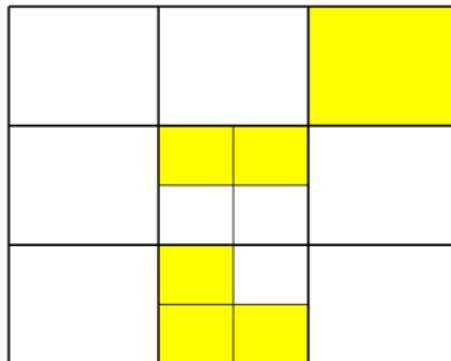
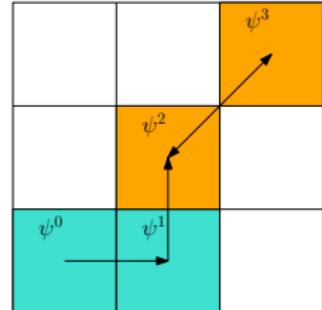


Transition $\psi^1 \rightarrow \psi^2$
 New valid set for ψ^1 : $V(\psi^1) \neq \emptyset$

Algorithm

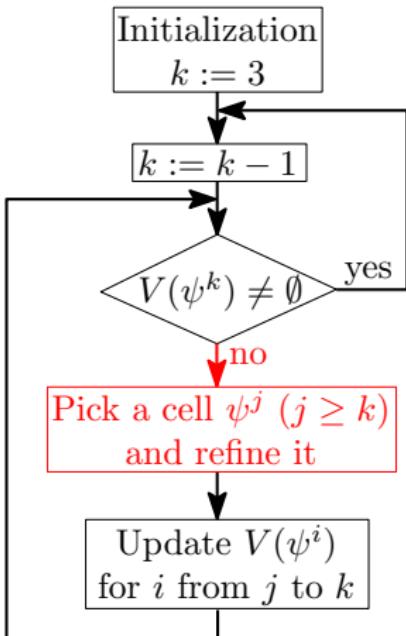


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

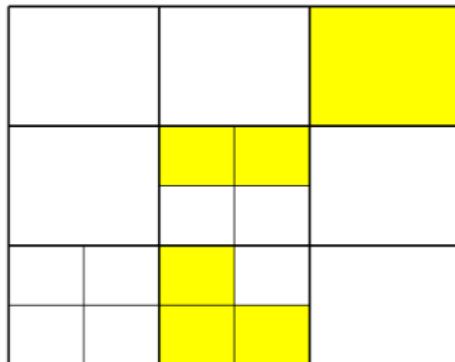
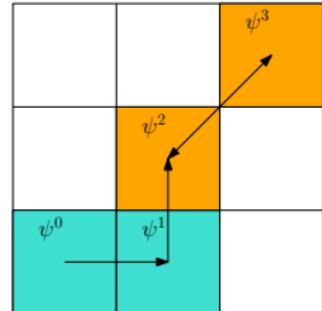


Transition $\psi^0 \rightarrow \psi^1$
Empty valid set: $V(\psi^0) = \emptyset$

Algorithm

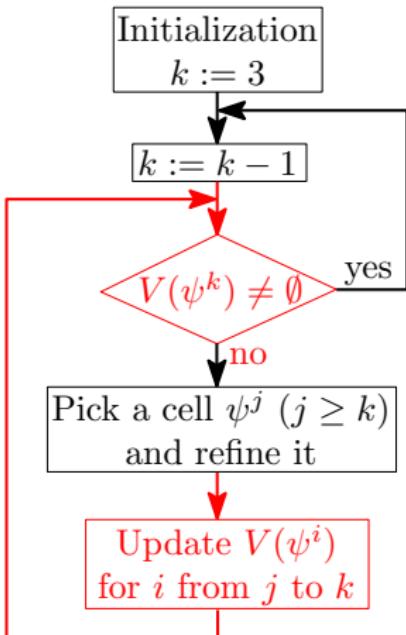


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

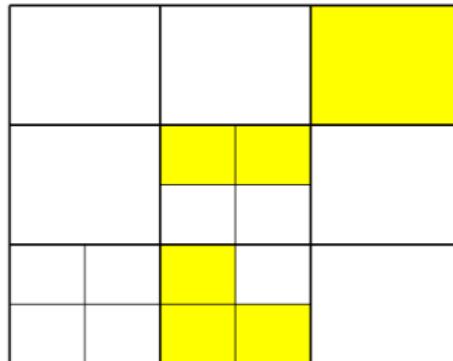
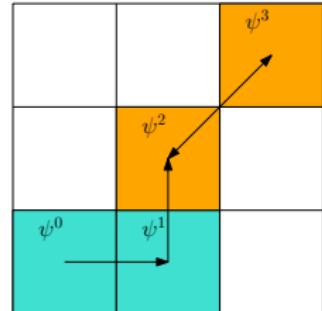


Transition $\psi^0 \rightarrow \psi^1$
 Refine ψ^0

Algorithm

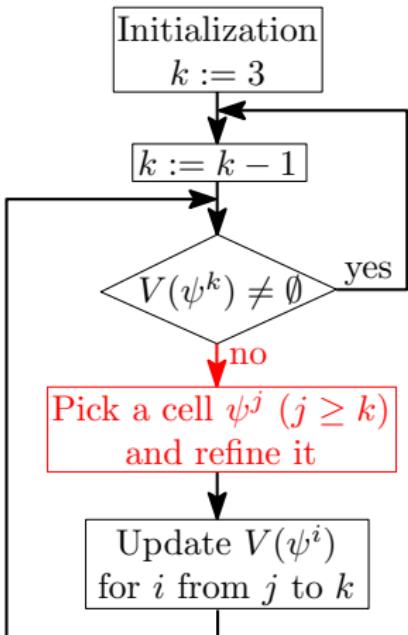


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

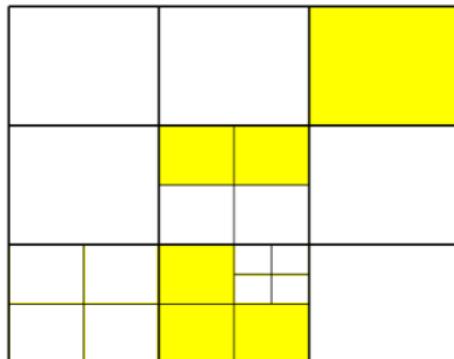
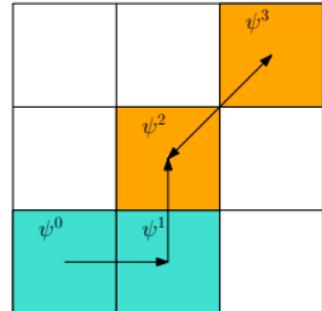


Transition $\psi^0 \rightarrow \psi^1$
Valid set still empty: $V(\psi^0) = \emptyset$

Algorithm

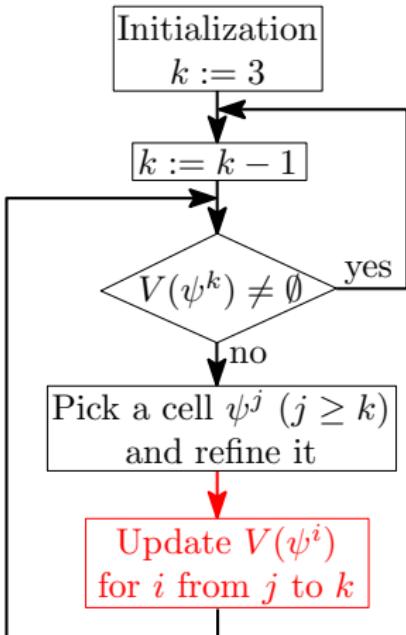


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

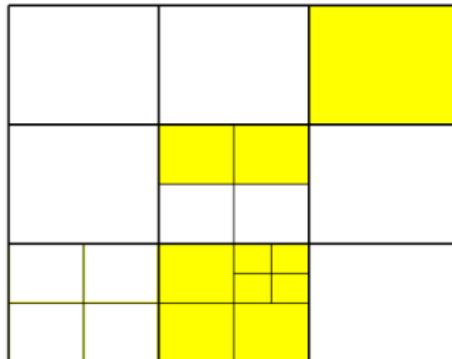
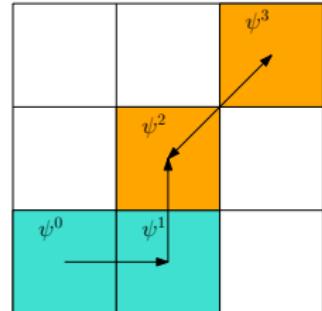


Transition $\psi^0 \rightarrow \psi^1$
Refine ψ^1

Algorithm

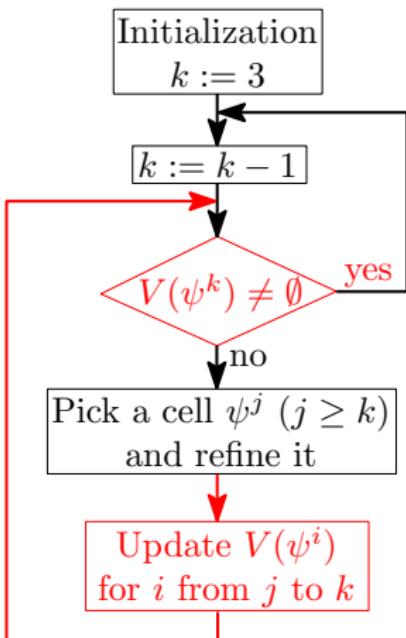


Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$

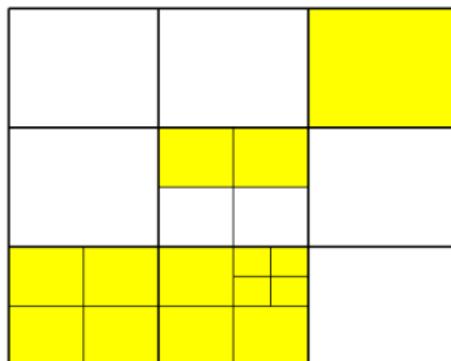
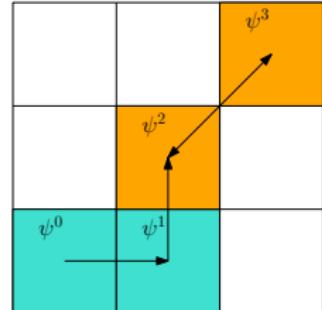


Transition $\psi^0 \rightarrow \psi^1$
 New valid set for ψ^1 : $V(\psi^1) \neq \emptyset$

Algorithm



Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$



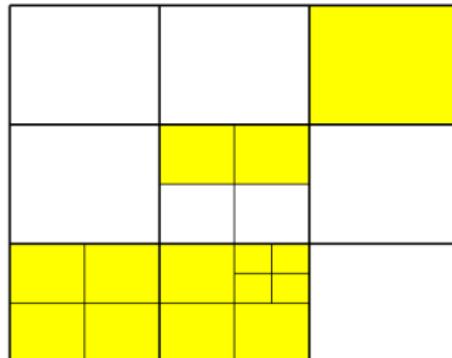
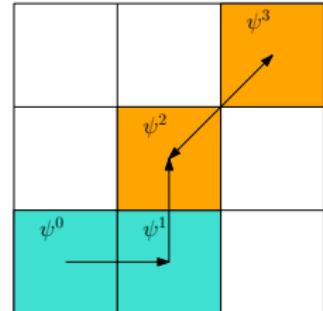
Transition $\psi^0 \rightarrow \psi^1$
 New valid set for ψ^0 : $V(\psi^0) \neq \emptyset$

Algorithm

Outputs

- ▶ **Refined partition**
- ▶ **Valid set** for each cell in the lasso
- ▶ **Controller** associated to each element of the valid sets

Specification:
 $\psi^0\psi^1(\psi^2\psi^3)^\omega$



Non-empty valid set for the initial cell: $V(\psi^0) \neq \emptyset$

Terminates the algorithm



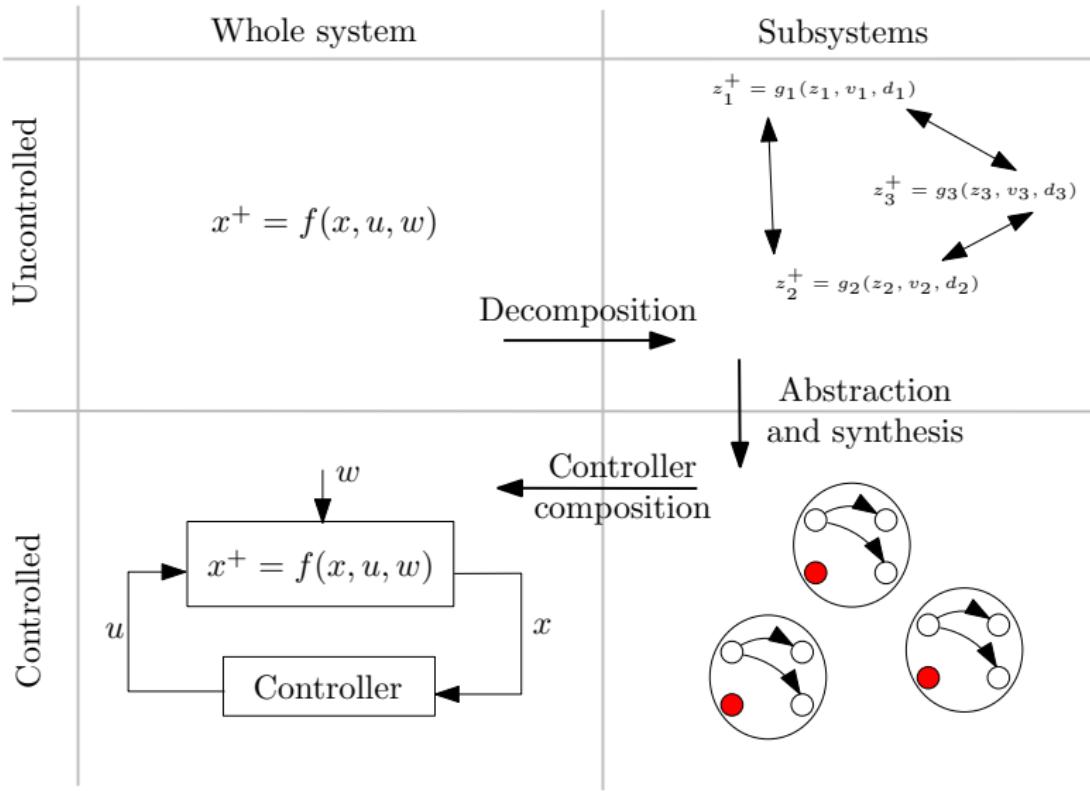
Outline

Context and intuition

Abstraction refinement algorithm

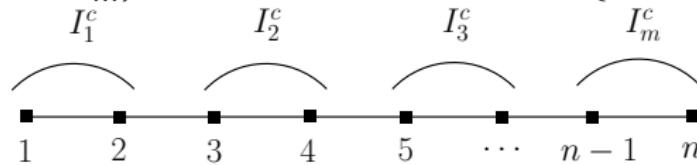
Compositional approach

Compositional synthesis

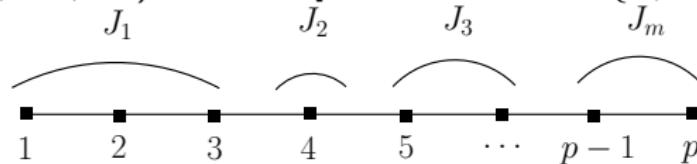


Decomposition into m subsystems

Partition (I_1^c, \dots, I_m^c) of the **state** dimensions $\{1, \dots, n\}$

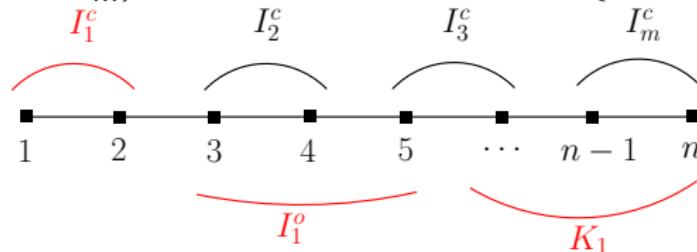


Partition (J_1, \dots, J_m) of the **input** dimensions $\{1, \dots, p\}$

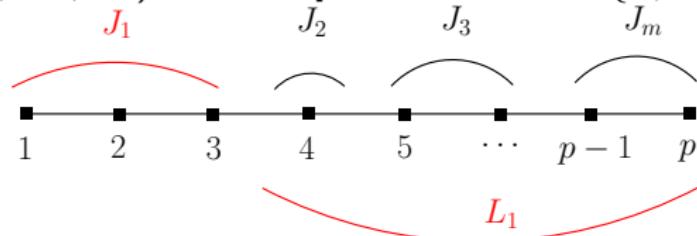


Decomposition into m subsystems

Partition (I_1^c, \dots, I_m^c) of the **state** dimensions $\{1, \dots, n\}$



Partition (J_1, \dots, J_m) of the **input** dimensions $\{1, \dots, p\}$



Goal: Control the states $x_{I_1^c}$ using the inputs u_{J_1} . States $x_{I_1^o}$ are observed by not controlled, x_{K_1} , and u_{L_1} act as disturbances.

Modeled states: $I_1 = I_1^c \cup I_1^o$



Compositional approach

Reachable set of subsystem i

$$RS_i(s_i, u_i) = RS(\psi^k \cap \pi_{I_i}^{-1}(s_i), U \cap \pi_{J_i}^{-1}(\{u_i\}))$$

- ▶ wider than $RS(s, u)$ due to **partial knowledge** of the global system



Compositional approach

Reachable set of subsystem i

$$RS_i(s_i, u_i) = RS(\psi^k \cap \pi_{I_i}^{-1}(s_i), U \cap \pi_{J_i}^{-1}(\{u_i\}))$$

- ▶ wider than $RS(s, u)$ due to **partial knowledge** of the global system

Compositional abstraction refinement

- ▶ **Decomposition** of the dynamics into subsystems
- ▶ Apply **abstraction refinement algorithm** to each subsystem i independently, using $RS_i(s_i, u_i)$
- ▶ Obtain refined partition X_i , valid set V_i and associated controller C_i for each subsystem i



Main result

Composition $C_c : X \rightarrow U$ of the local controllers:

$$C_c(x) = (C_1(x_{I_1}), \dots, C_m(x_{I_m})), \forall x \in X$$

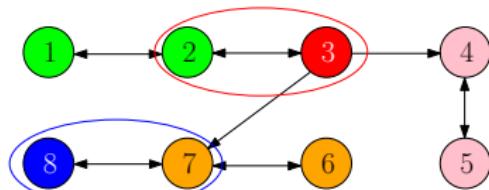
Theorem

If the abstraction refinement **algorithm terminates for all subsystems**, then the closed-loop system $x^+ = f(x, C_c(x), w)$ follows the specification sequence ψ

Compositional example

$$\dot{x} = A * x - \beta * x^3 + u$$

► $x, u \in \mathbb{R}^8$



$$I_1^c = J_1 = \{1, 2\}$$

$$I_1^o = \emptyset$$

$$I_2^c = J_2 = \{4, 5\}$$

$$I_2^o = \emptyset$$

$$I_3^c = J_3 = \{6, 7\}$$

$$I_3^o = \emptyset$$

$$I_4^c = J_4 = \{3\}$$

$$I_4^o = \{2\}$$

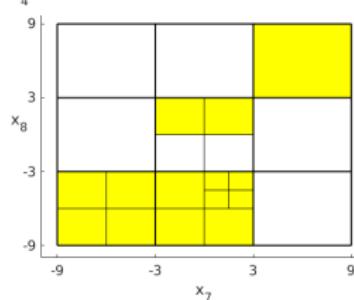
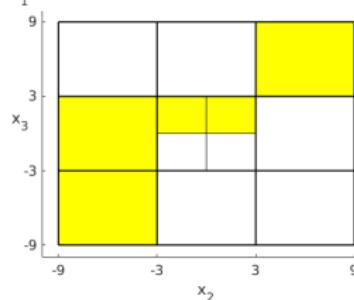
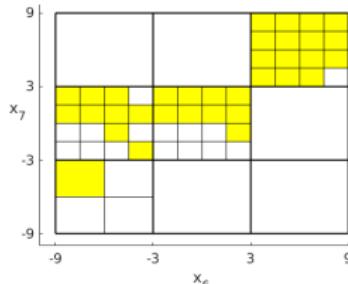
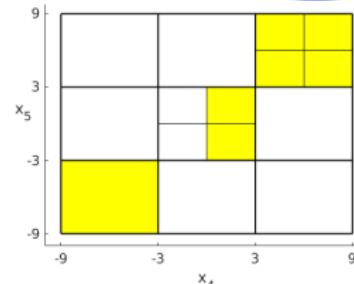
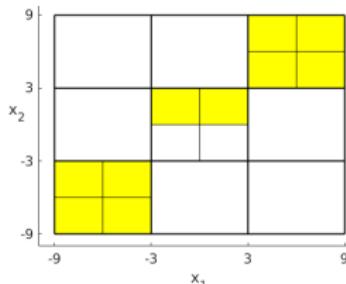
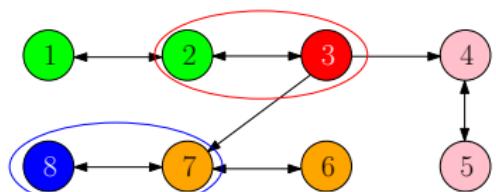
$$I_5^c = J_5 = \{8\}$$

$$I_5^o = \{7\}$$

Compositional example

$$\dot{x} = A * x - \beta * x^3 + u$$

- ▶ $x, u \in \mathbb{R}^8$
- ▶ Computations: 11s





Conclusion and perspectives

Contributions

- ▶ abstraction refinement algorithm for non-deterministic abstractions with lasso-shaped specification
- ▶ compositional refinement framework

Degrees of freedom

- ▶ splitting strategy when refining
- ▶ order in which the cells are refined

Perspectives

- ▶ obtain the lasso sequence from a Linear Temporal Logic formula
- ▶ combine abstraction refinement with plan revision



Assume-Guarantee Obligations

Assume-guarantee approach: internal assumptions that all other subsystems correctly realize their part of the specification

A/G Obligation (Unobserved states K_i)

For all subsystem $i \in \{1, \dots, m\}$, for all $x \in X$, for all cell ψ^k

$$x_{I_i} \in \pi_{I_i}(\psi^k) \implies x_{K_i} \in \pi_{K_i}(\psi^k)$$

A/G Obligation (Observed but uncontrolled states I_i^o)

For all subsystem $i \in \{1, \dots, m\}$, for all $x \in X$, for all cell ψ^k

$$x_{I_i} \in \pi_{I_i}(\psi^k) \implies \pi_{I_i^o}(f(x, u, w)) \in \pi_{I_i^o}(\psi^k)$$