Using progress sets on non-deterministic transition systems for multiple UAV motion planning

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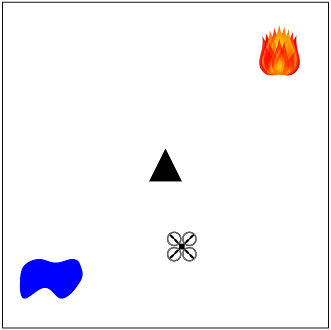
Outline

Context and motivation

Progress sets

Experimental results

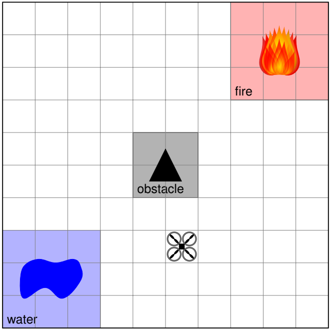
Motivation example



High-level control objective:

- get some water to be dropped on the fire
- while avoiding the obstacle

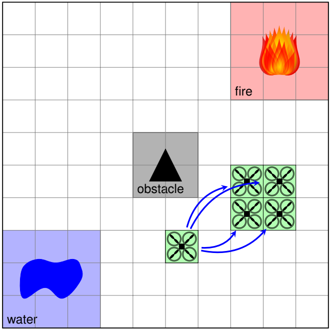
Discrete representation



Partition of the environment

- relevant cells labeled with water, fire and obstacle
- other cells unlabeled

Discrete representation

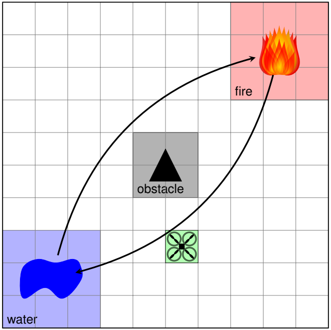


Model the system as a **Finite Transition System**

Non-determinism caused by:

- disturbances
- unknown initial state within a cell

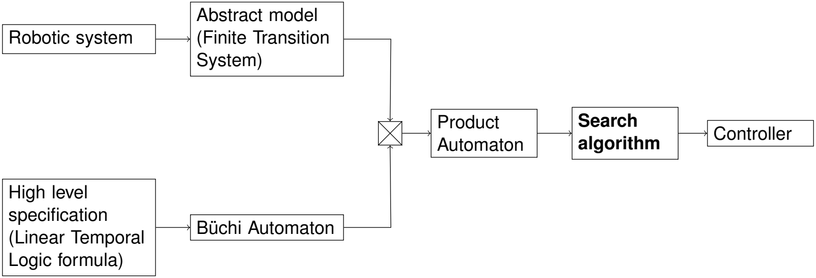
High-level specifications



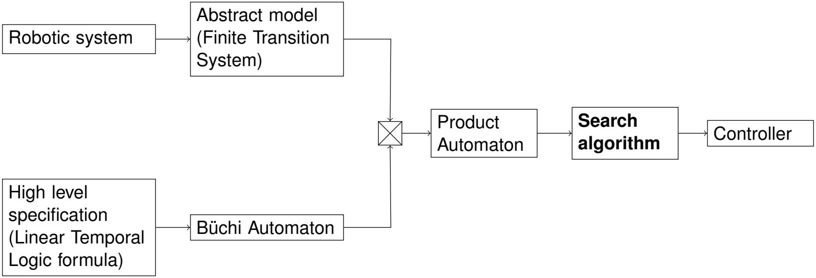
Linear Temporal Logic formula

- $\varphi = (\Box\Diamond \mathit{water}) \wedge (\Box\Diamond \mathit{fire}) \wedge (\Box\neg \mathit{obstacle})$
- goal: find a controller such that the formula is satisfied by the sequence of labels generated by the controlled system
- label sequence: $\{\emptyset\}, \ldots, \{\emptyset\}, \{\mathit{water}\}, \{\emptyset\}, \ldots, \{\emptyset\}, \{\mathit{fire}\}, \ldots$

Control synthesis approach



Control synthesis approach

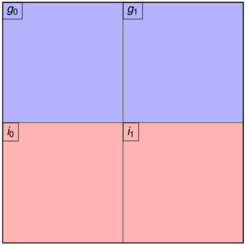


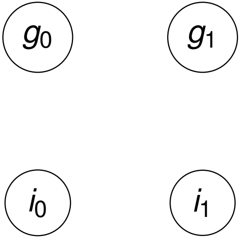
For **non-deterministic** transition systems, finding a controller may not be possible.

- Consider **augmented** transition systems with **progress sets** that represent guarantees of progress towards satisfaction of the specification

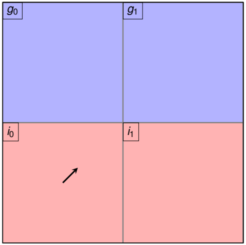
Outline

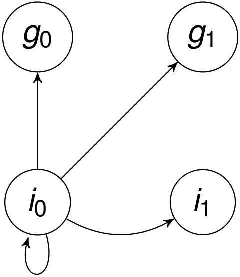
Progress set: intuition





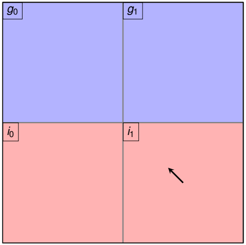
Progress set: intuition

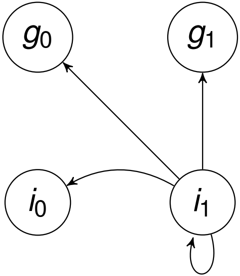




Possible transitions from i_0 with control input $\nearrow$

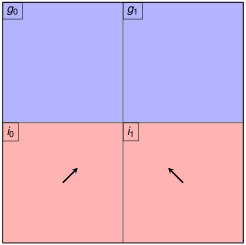
Progress set: intuition

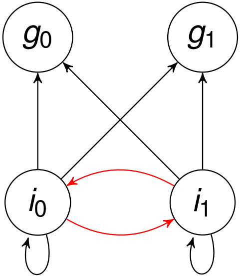




Possible transitions from i_1 with control input $\nwarrow$

Progress set: intuition

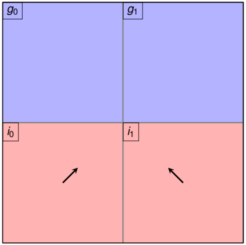


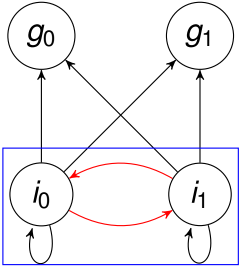


Possible infinite behavior on the transition system:

$$i_0 \xrightarrow{\nearrow} i_1 \xrightarrow{\nwarrow} i_0 \xrightarrow{\nearrow} i_1 \xrightarrow{\nwarrow} i_0 \xrightarrow{\nearrow} i_1 \xrightarrow{\nwarrow} \dots$$

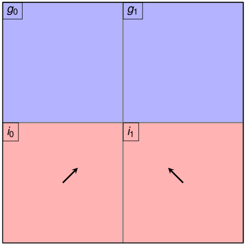
Progress set: intuition

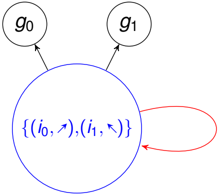




This infinite behavior may not be actually feasible by the continuous dynamics if i_0 and i_1 are considered together

Progress set: intuition

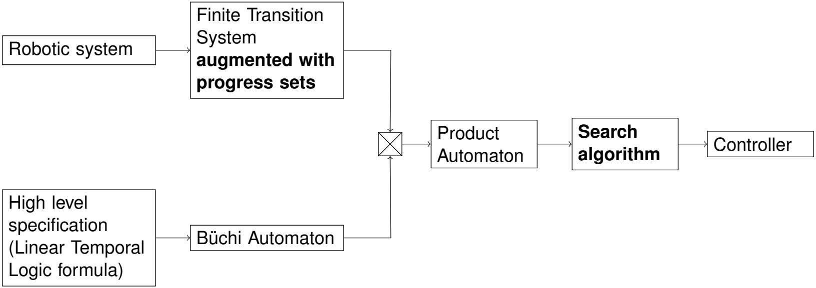




Progress set: set $\{(q_1, u_1), \ldots, (q_m, u_m)\} \in 2^{Q \times U}$ of pairs (state,control) whose combined action is guaranteed to eventually leave the corresponding set of states $\{q_1, \ldots, q_m\}$.

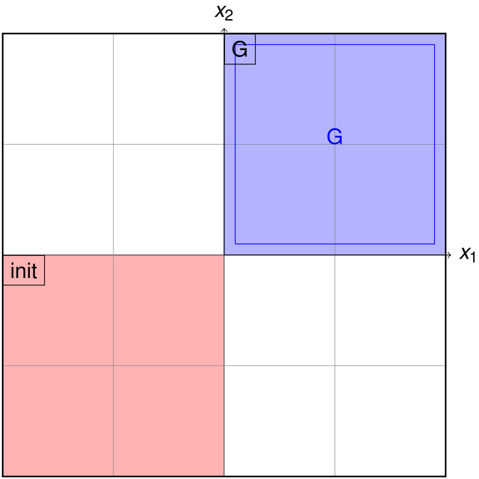
Nilsson and Ozay, *Incremental synthesis of switching protocols via abstraction refinement*, CDC 2014.

Control synthesis approach



- Provided: **augmented transition system** with progress sets
- **Main contribution**: how to use these progress sets for the control synthesis

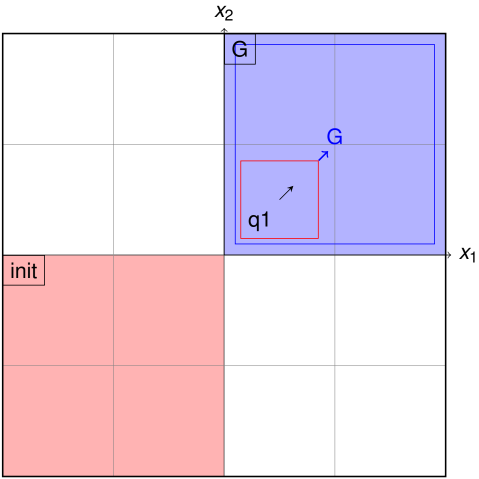
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
G

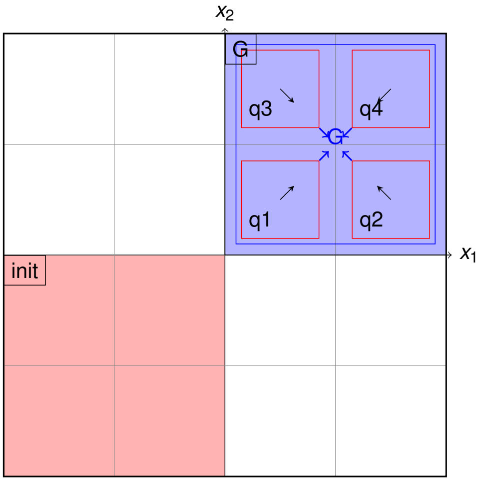
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_1 = G$

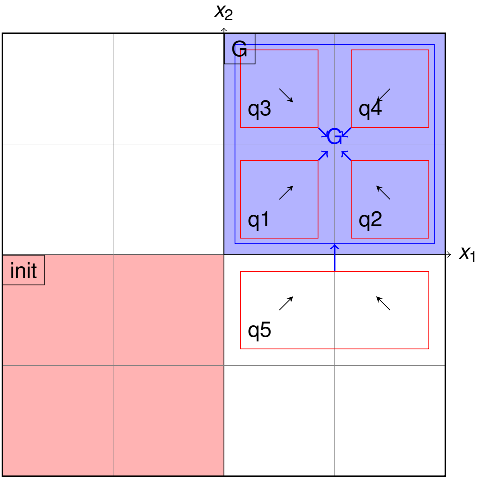
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_2 \cup q_3 \cup q_4 = G$

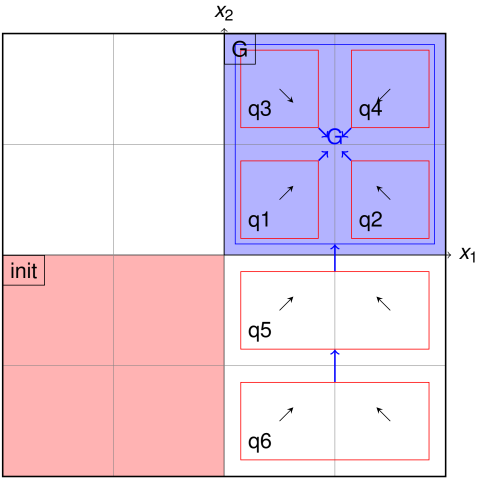
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_5$

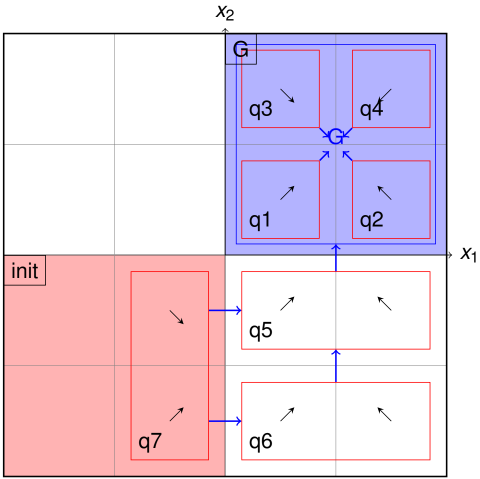
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_5 \cup q_6$

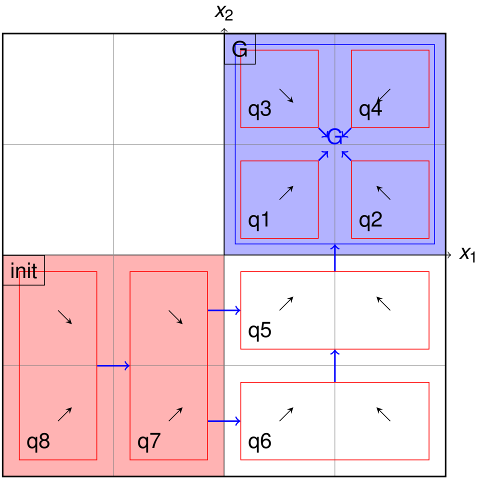
Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_5 \cup q_6 \cup q_7$

Search algorithm using progress sets



Expand goal set G with the progress sets guaranteed to go to G

Goal set:
$G \cup q_5 \cup q_6 \cup q_7 \cup q_8$

Algorithm terminates:
$init \subseteq G \cup q_5 \cup q_6 \cup q_7 \cup q_8$

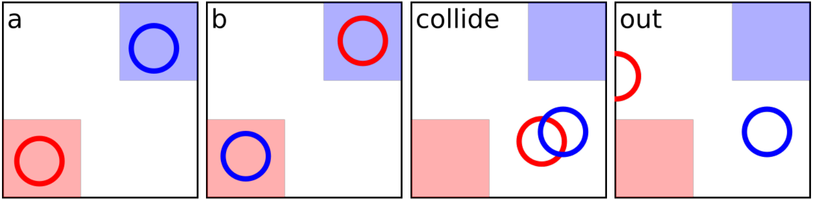
Outline

Experiment

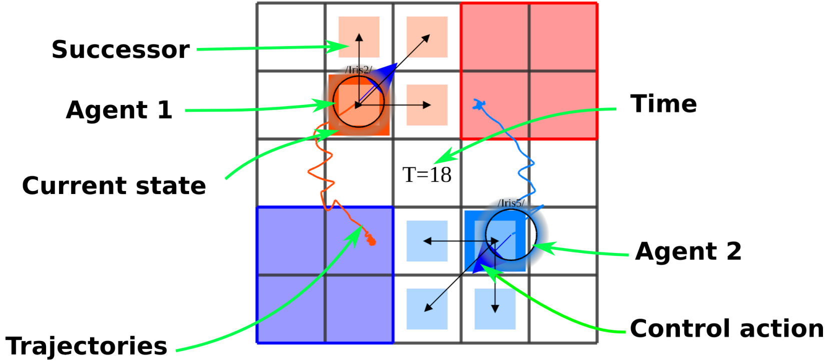
System constituted of 2 quadrotors: quad 1 and quad 2

Control objective: surveillance and safety

- $\varphi = (\Box\Diamond a) \wedge (\Box\Diamond b) \wedge (\Box\neg collide) \wedge (\Box\neg out)$



Experiment - video



Conclusion

Proposed approach

- Consider **non-deterministic** finite transition systems augmented with **progress sets**
- Propose a new planning algorithm for the control synthesis under LTL specifications

Future work

- Computation of progress sets for general classes of systems

Progress set identification

Single integrator system with disturbances:

- $\dot{x} = u + w$

Abstract continuous dynamics into a Finite Transition System

- Q: finite set of states
- U: finite set of control inputs

Check a candidate progress set

- candidate: $p = \{(q_0, u_0), (q_1, u_1), ..., (q_n, u_n)\} \in 2^{Q \times U}$
- get the set of controls $U = \{u_0, u_1, ..., u_n\}$ involved in p
- compute the convex hull $\mathcal{C}_U$ of U

$$p \text{ is a progress set} \Leftrightarrow \mathbf{0} \notin \mathcal{C}_U$$