KTH ROYAL INSTITUTE
OF TECHNOLOGY

Multi agent control with LTL specifications and abstraction with input memories

Paul Rousse

Automatic Departement

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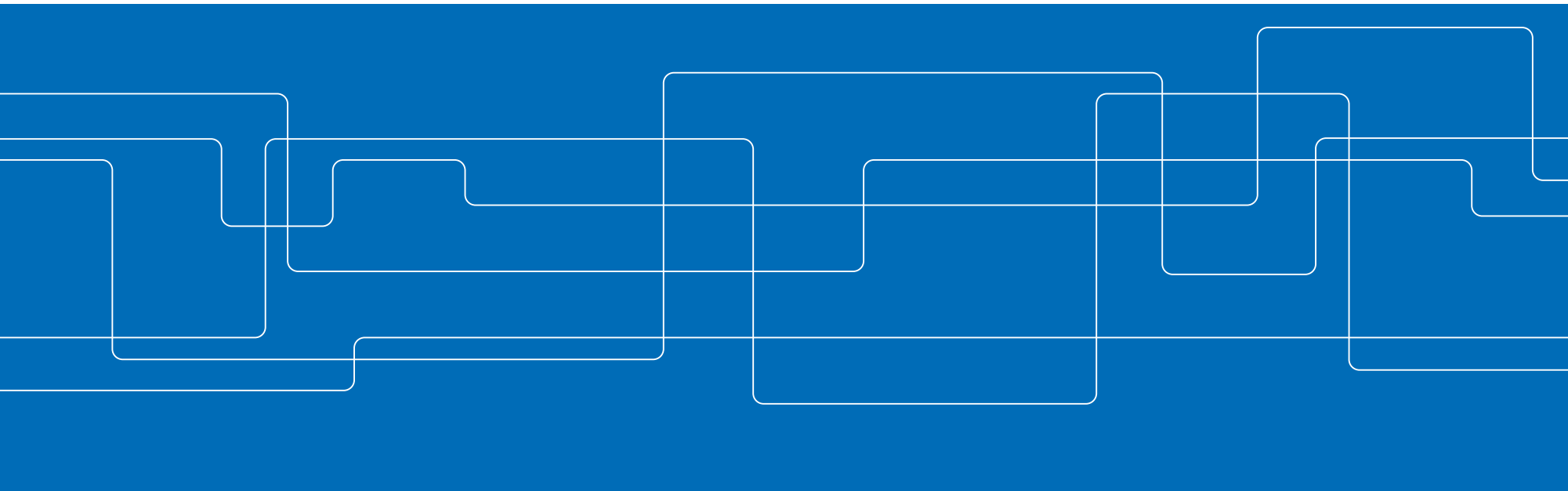
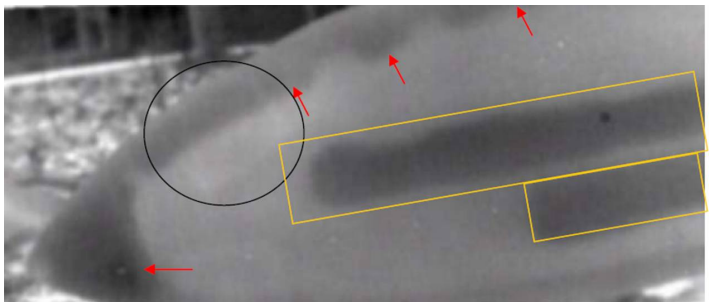


Wind turbine maintenance scenario



Wind turbine maintenance scenario

- Motion and task specifications
- Safety requirements (avoid collision, stay inside a safety area)

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$\Rightarrow$ Linear Temporal Logic (LTL)

$\Rightarrow$ formal controller synthesis

Outline

High level specification: the temporal logic

Linear temporal logic (LTL)

$$\varphi ::= \top \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

Temporal operators:

- $\bigcirc$ next
- U until
- $\Box$ always
- $\Diamond$ eventually

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Temporal operators:

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$$\varphi = \Box\Diamond a$$

$$\begin{aligned} \sigma_1 &= 111a111a\overline{111a} \\ \Longrightarrow \sigma_2 &= 111a111a111\overline{1} \\ \sigma_3 &= 11111111111\overline{1} \end{aligned}$$

Büchi Automaton

Nondeterministic Büchi Automaton

$\mathcal{A}_\varphi = (Q, Q_0, 2^{AP}, \delta, \mathcal{F})$ where:

- Q finite set of states;
- $Q_0 \subset Q$ a set of initial states;
- 2^{AP} the alphabet;
- $\delta : Q \times 2^{AP} \times Q$ a transition relation ;
- $\mathcal{F} \subseteq Q$ set of accepting states.

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$$\sigma = l_1, l_2, l_3, ... \rightarrow q_0, q_1, q_2, ...$$

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Acceptance condition: $Inf(\{q_0, q_1, q_2, ...\}) \cap \mathcal{F} \neq \emptyset$

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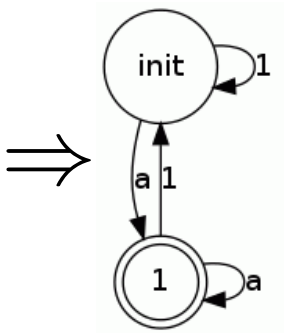
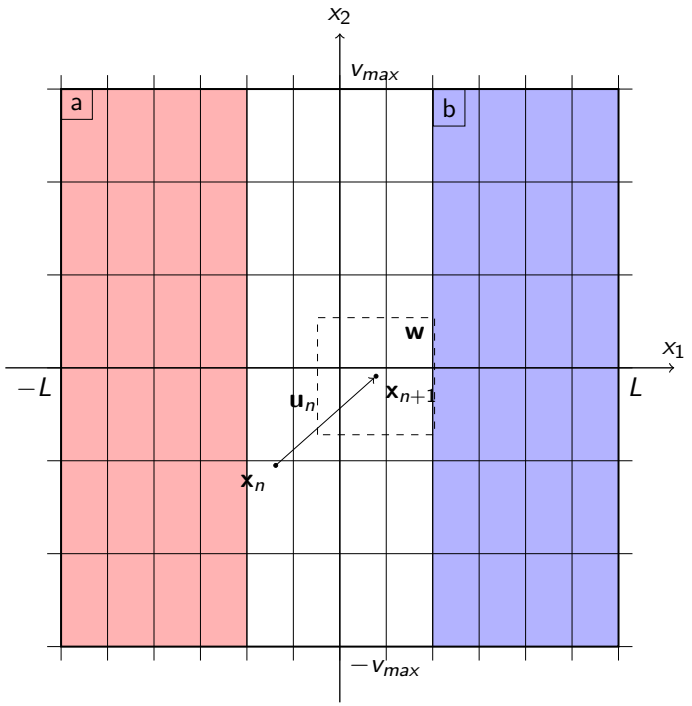


Abstraction



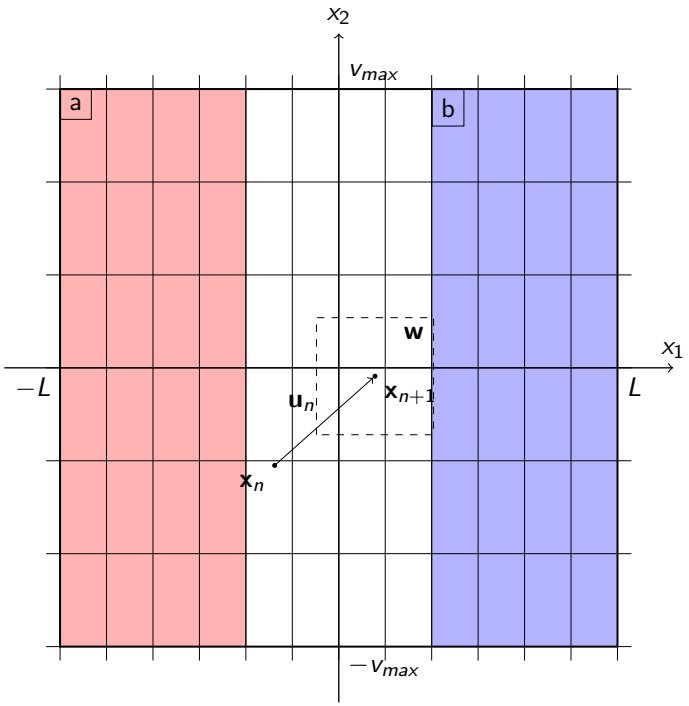
$\rightarrow$

$\mathcal{T}_c = (X, X_0, \mathcal{U}, Post_{\mathbf{u}}, H)$

- X states
- $X_0 \subseteq X$ initial states
- U input set
- $Post_{\mathbf{u}} \subseteq X \times U \times X$ transition relation
- $H : X \rightarrow Y$ output map

Figure : Model of the robot

Abstraction



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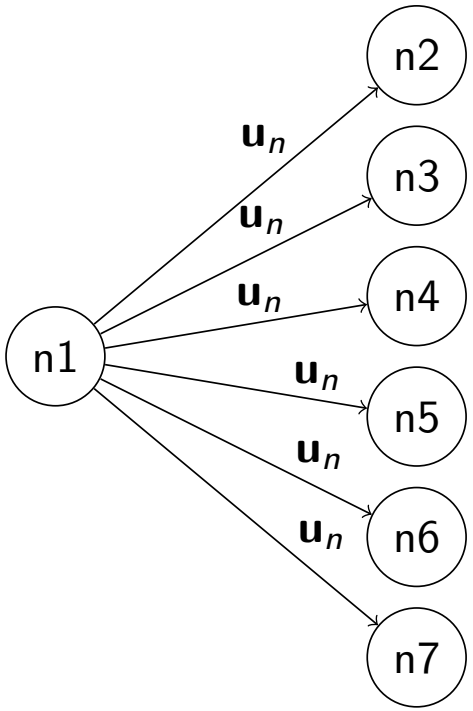


Figure : Finite Transition System

Controller synthesis

Product of a NBA and a FTS

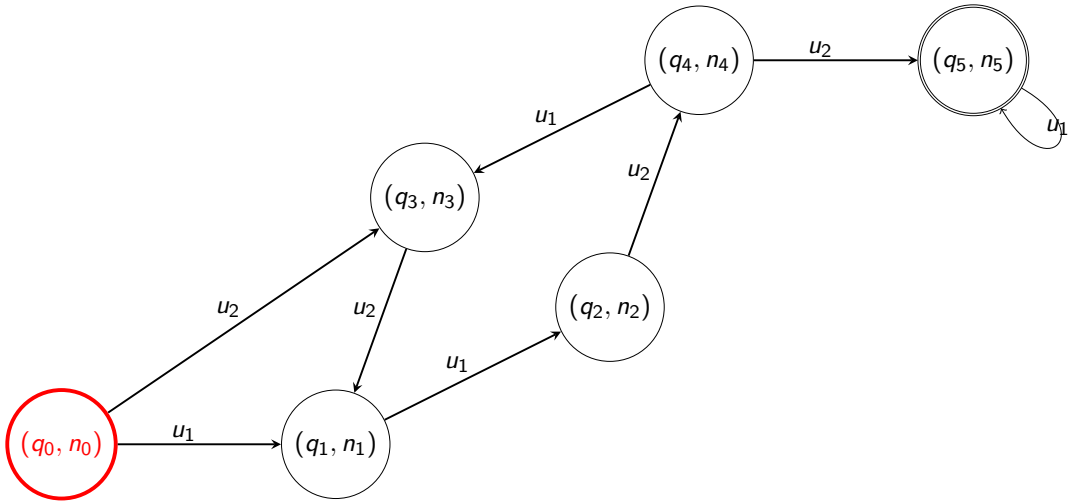
$\mathcal{A}_p = \mathcal{T}_c \otimes \mathcal{A}_\varphi = (Q', Q'_0, \delta', \mathcal{F}')$

- $Q' = X \times Q$ is the set of states,
- $Q'_0 = X_0 \times Q_0$ is the set of initial states,
- $\mathcal{F}' = X \times \mathcal{F}$ the acceptance set,
- $\delta' \subseteq Q' \times U \times Q'$ is the transition relation defined by $\langle x', q' \rangle \in \delta'(\langle x, q \rangle, u)$ iff $x' \in Post_u(x)$, $q' \in \delta(q, L_c(x'))$ and

$$\forall h \in H(Post_u(x)), \exists y \in H^{-1}(h) \cap Post_u(x), \delta(q, L_c(y)) \neq \emptyset$$

Controller synthesis

Use your favourite graph search algorithm (Depth/Breadth First Search, Fixed points...)



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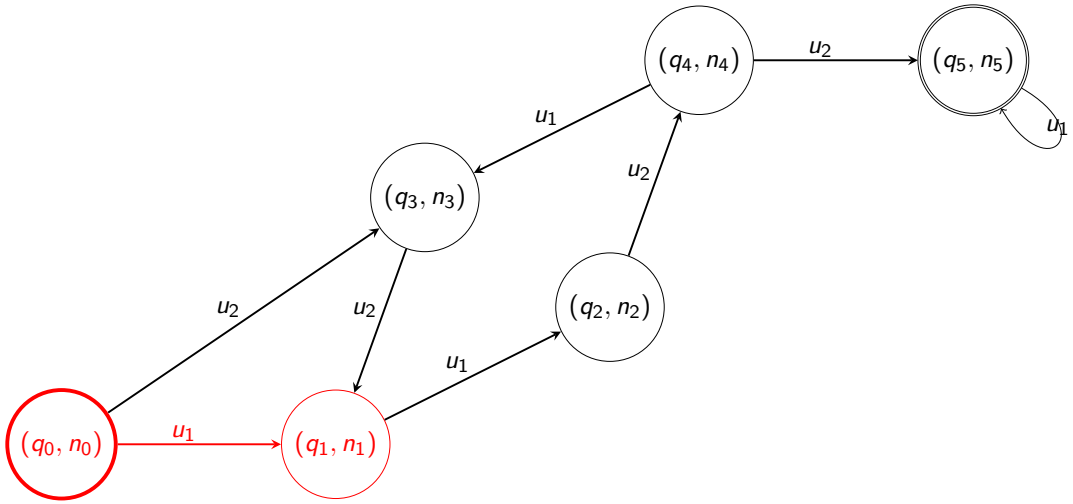

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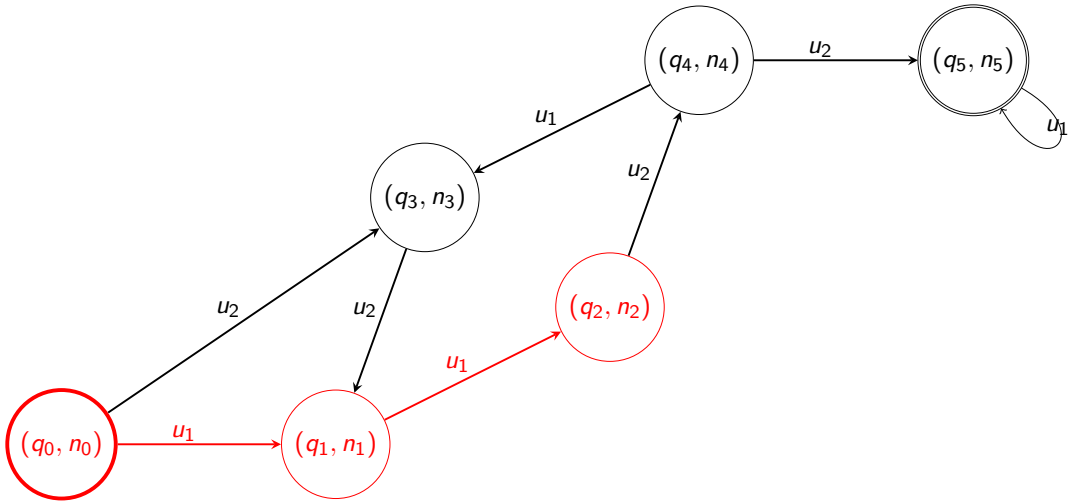

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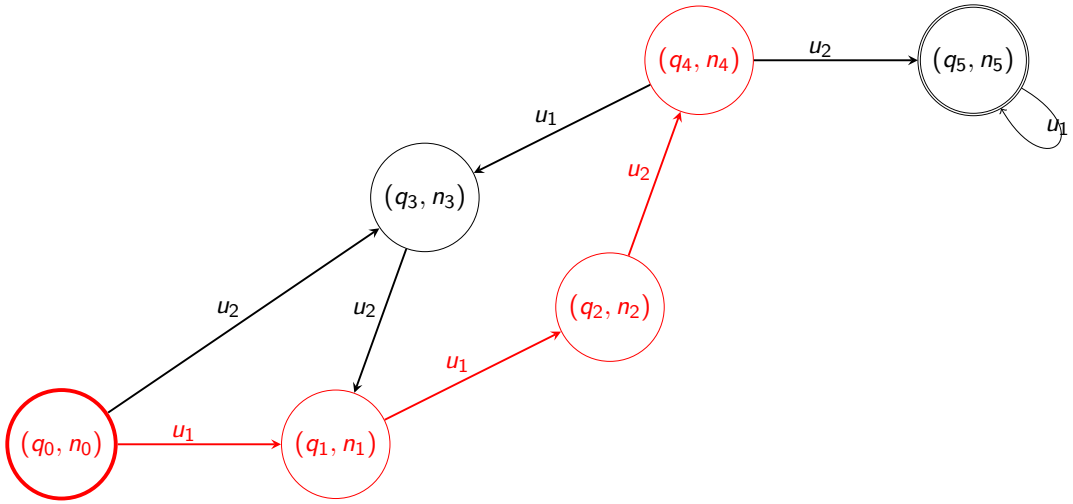

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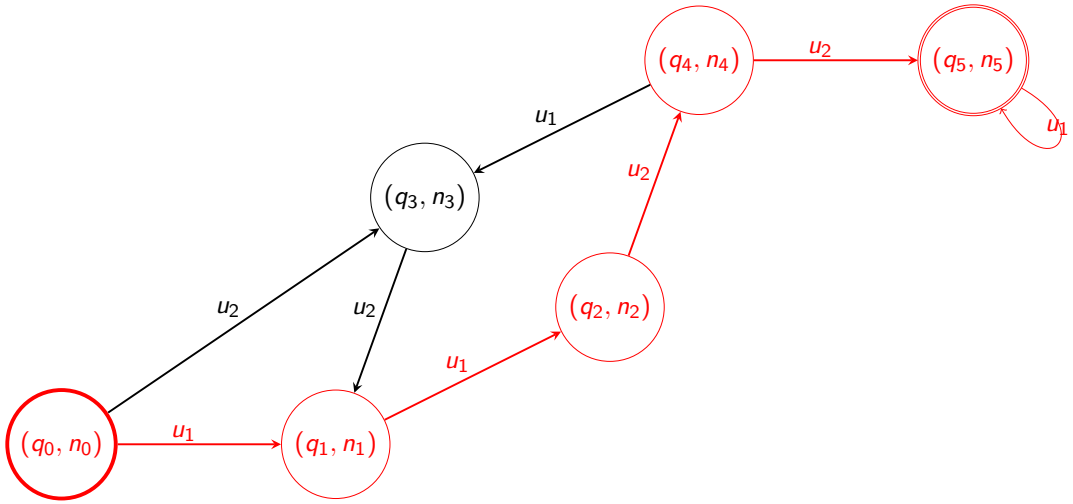

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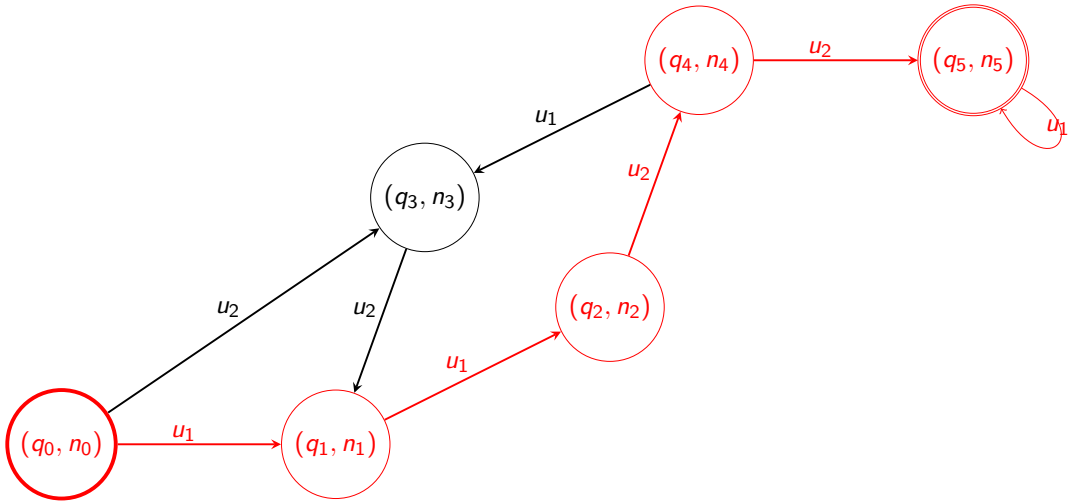

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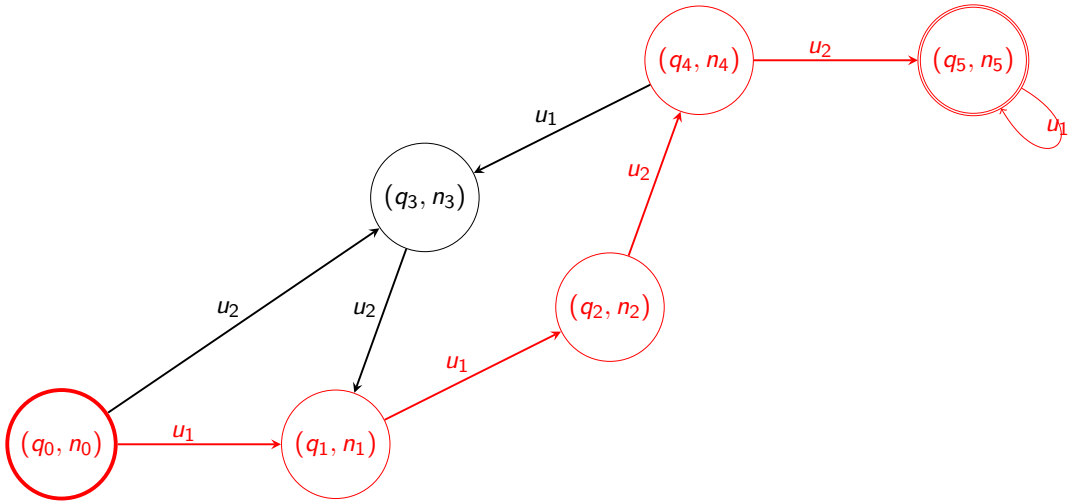

$\Rightarrow$ Controller $=$
$\{((q_1, n_1), u_1), ((q_2, n_2), u_2), \ldots, ((q_n, n_n), u_n)\}$

Recap

Model of the robot

Controller

High level specifications

Recap


Model of the robot
Controller
High level specifications
Büchi Automaton

Recap


Model of the robot
Abstraction (FTS)
Controller
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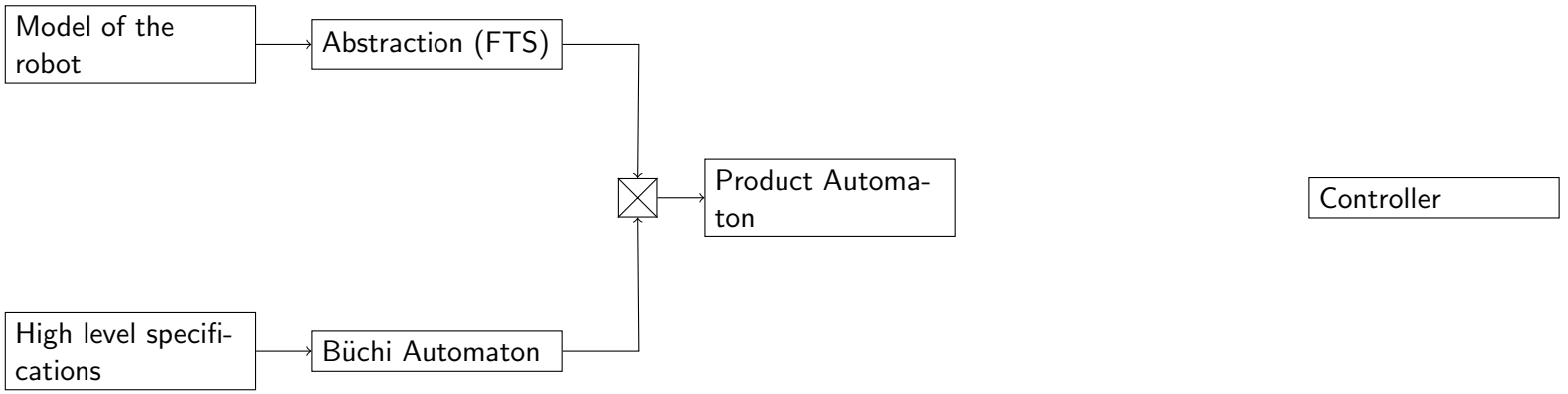
Recap


Model of the robot
Abstraction (FTS)
Product Automaton
Controller
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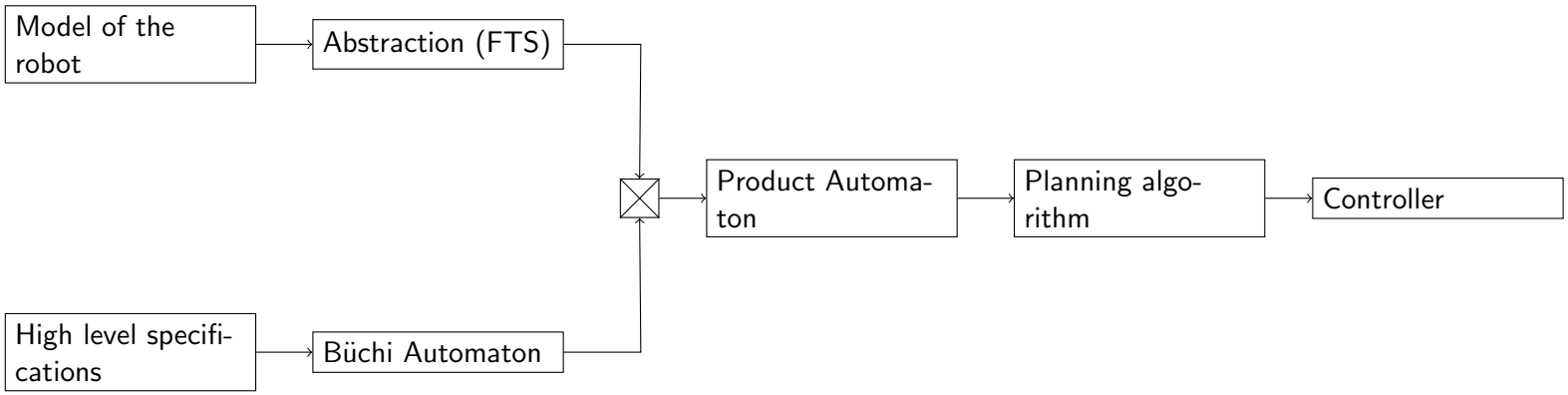
Recap


Model of the robot
Abstraction (FTS)
High level specifications
Büchi Automaton
Product Automaton
Planning algorithm
Controller

State extended abstraction

Dynamical system:

- state: $(\mathbf{x}^{obs}, \mathbf{x}^{unobs})$
- transition: $(\mathbf{x}^{obs}, \mathbf{x}^{unobs}) \xrightarrow[S]{\mathbf{u}} (\mathbf{x}_+^{obs}, \mathbf{x}_+^{unobs})$

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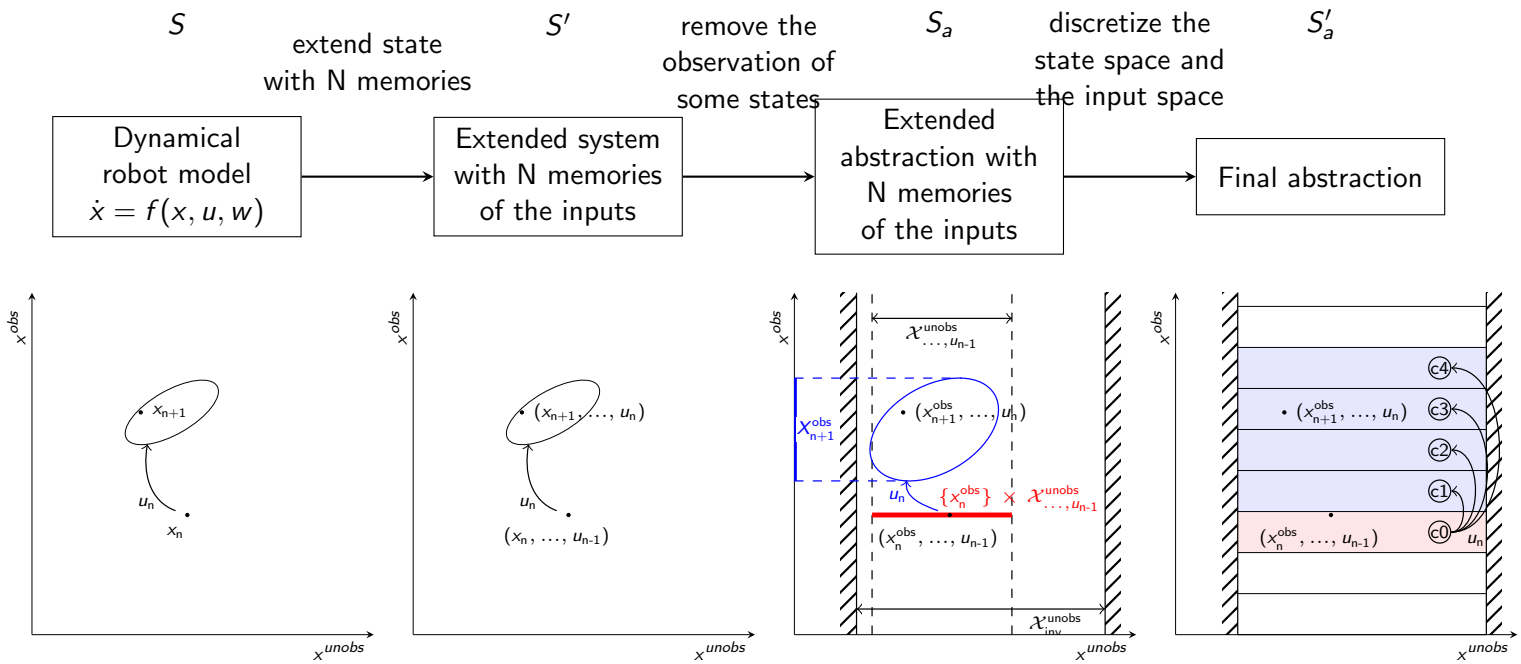
Abstraction:

- $(\mathbf{x}^{obs}, \mathbf{u}_{n-N}, \ldots, \mathbf{u}_{n-1})$
- transition:
  $(\mathbf{x}^{obs}, \mathbf{u}_{n-N}, \ldots, \mathbf{u}_{n-1}) \xrightarrow[S_a]{\mathbf{u}} (\mathbf{x}_+^{obs}, \mathbf{u}_{n+1-N}, \ldots, \mathbf{u}_{n-1}, \mathbf{u})$

State extended abstraction



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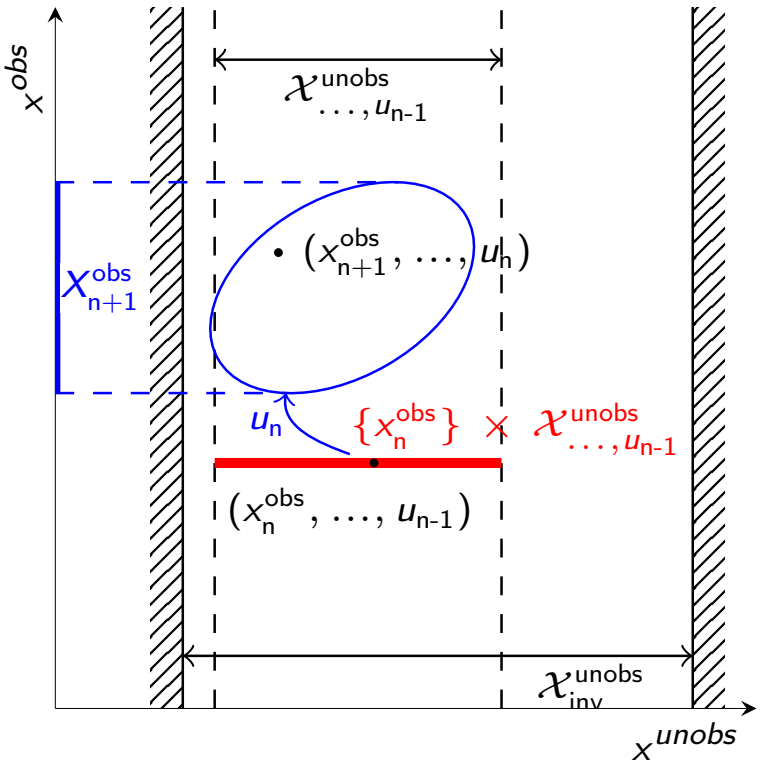

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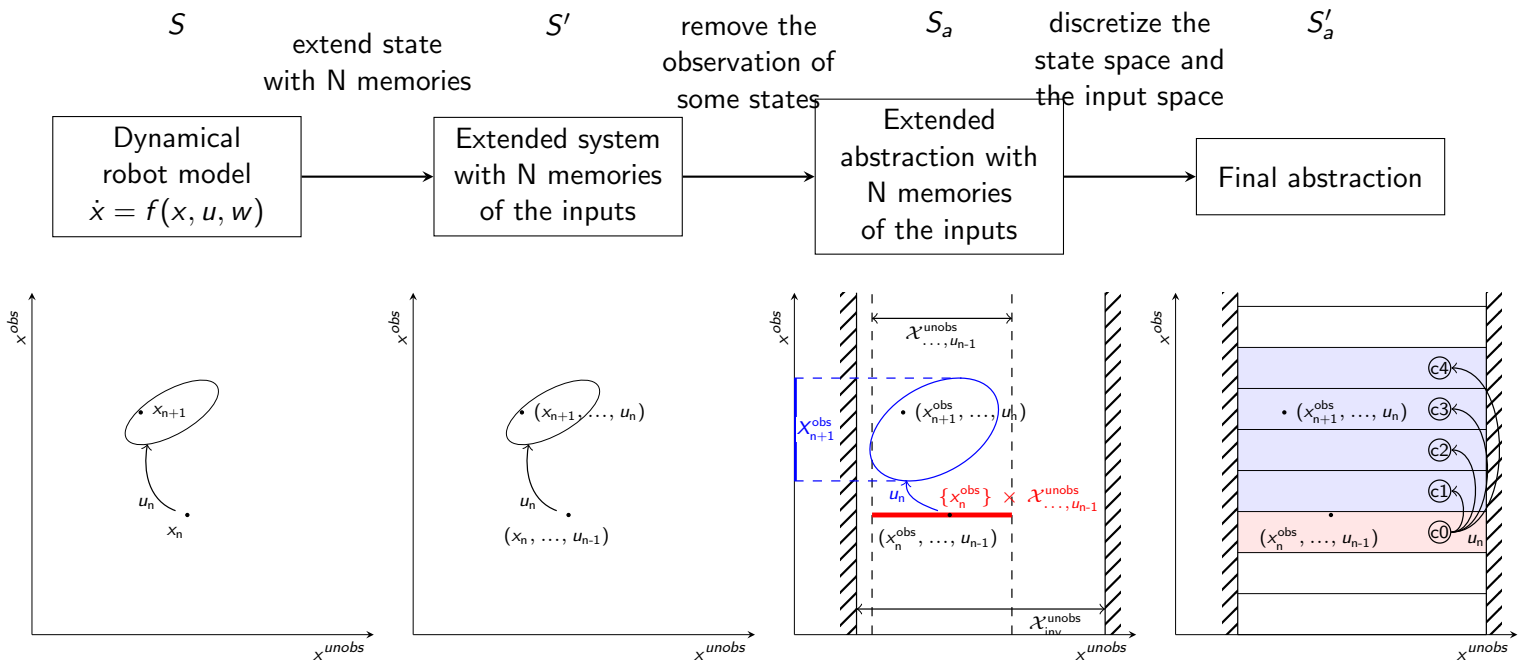
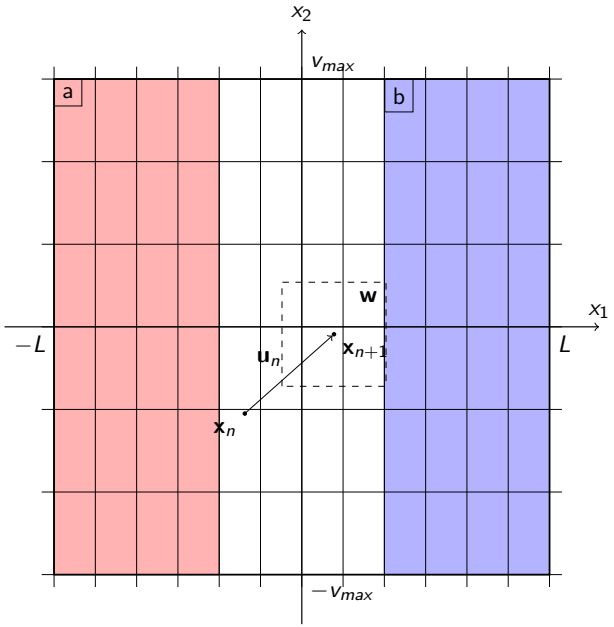
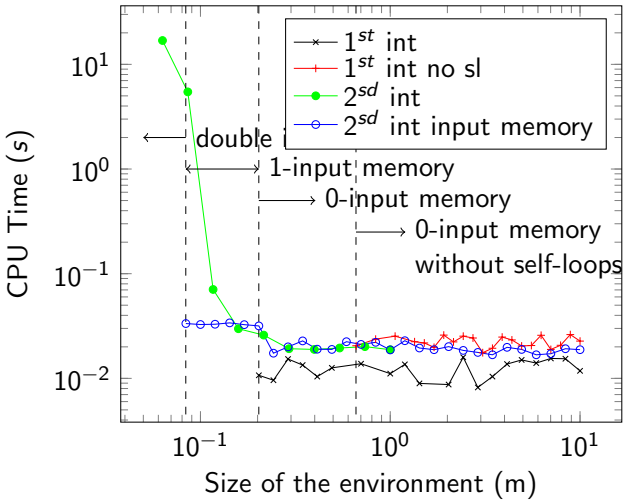


Comparison with uniform discretization



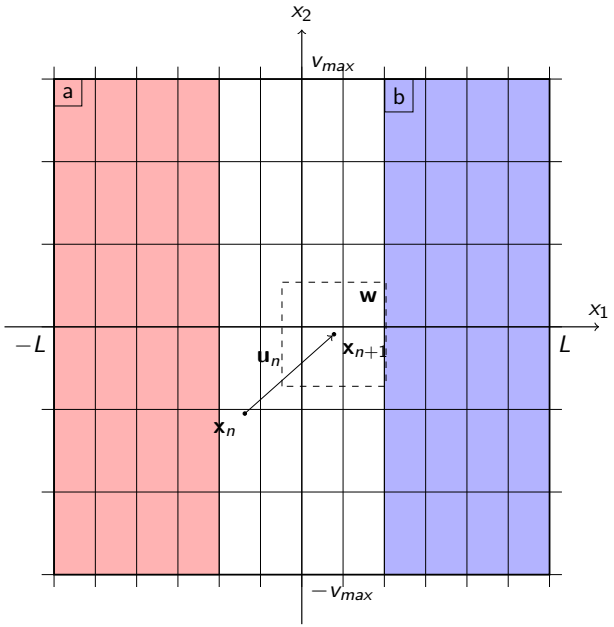


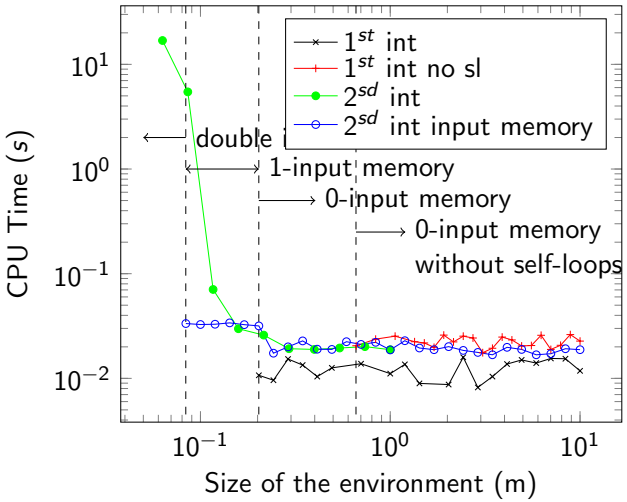
$$\dot{\mathbf{v}} = k(\mathbf{v}_{ref} - \mathbf{v}) + \mathbf{w}_v$$

$$\dot{\mathbf{x}} = \mathbf{v} + \mathbf{w}_x$$

$$\varphi = (\square\diamond a) \wedge (\square\diamond b)$$

Comparison with uniform discretization





- more conservative (the information is selected more wisely)

Admissible noise

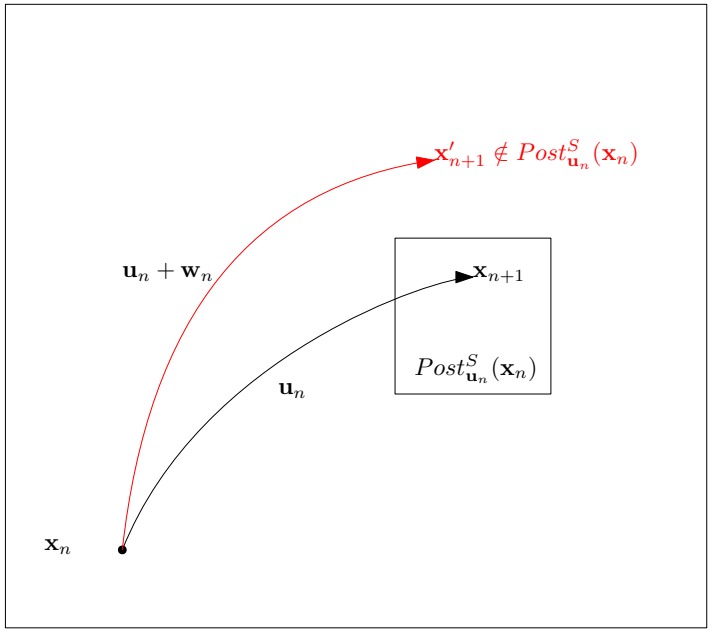
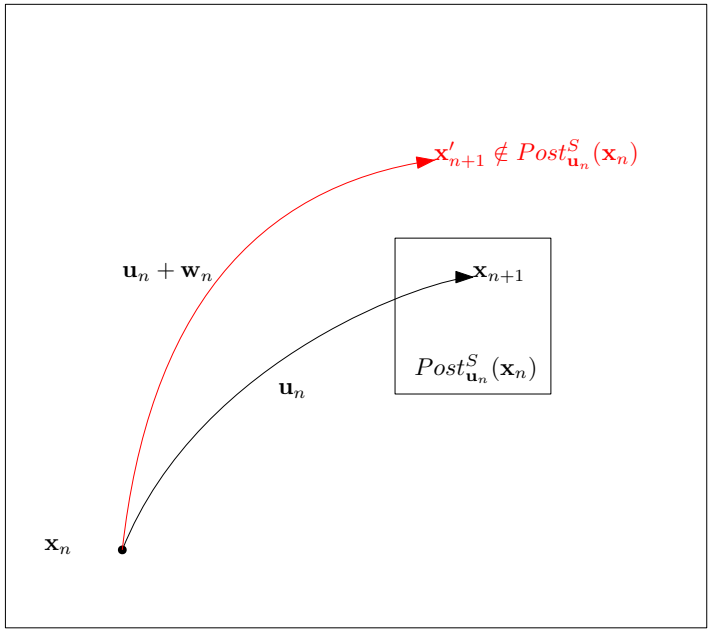


Figure : Dynamical system

Admissible noise



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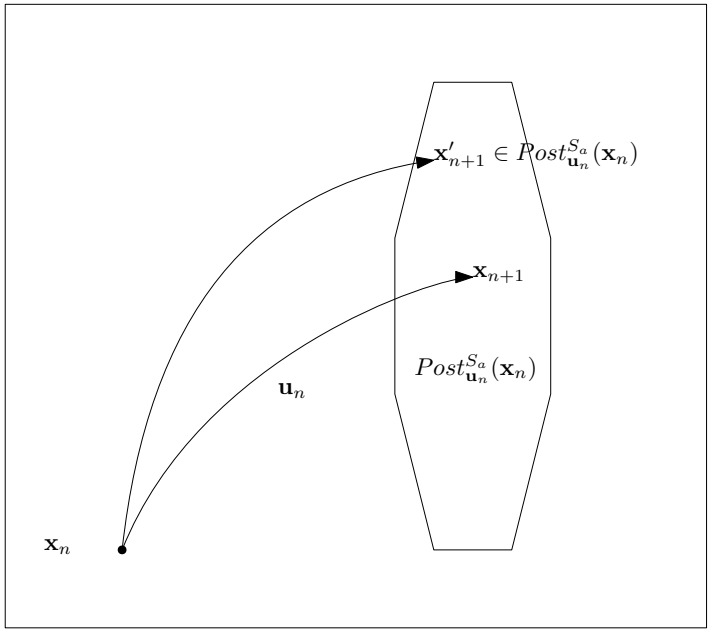
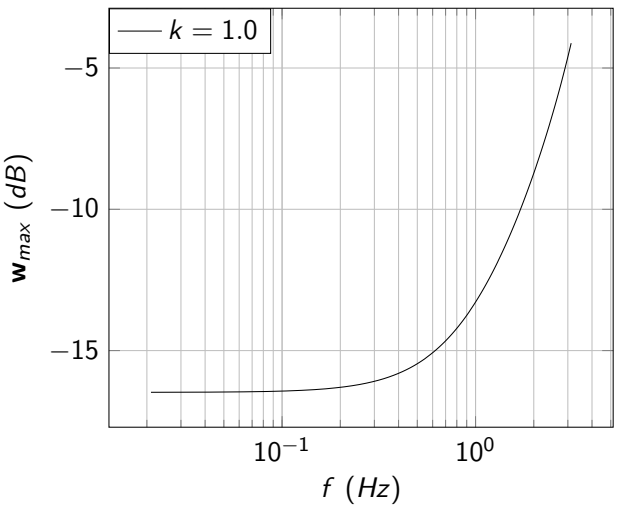
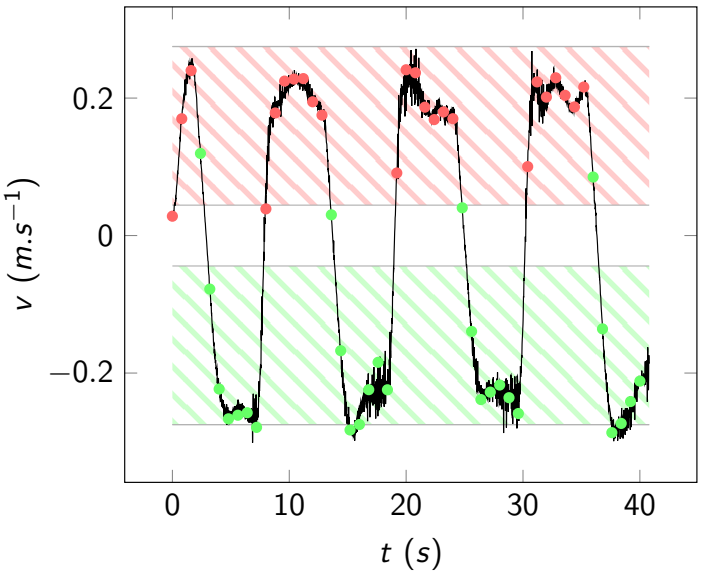


Figure : Abstraction

Admissible noise

$$\mathcal{W}_a = \{\{\mathbf{w}_n\}_{n\in\mathbb{N}} \mid \mathcal{L}_1(G^{unobs} * G_{\mathbf{w}}) \leq \sigma\}$$
$$\mathcal{W} \subset \mathcal{W}_a$$





Path planning algorithm

Planner specifications:

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$\Rightarrow$ Strong cyclic planner with local fairness property

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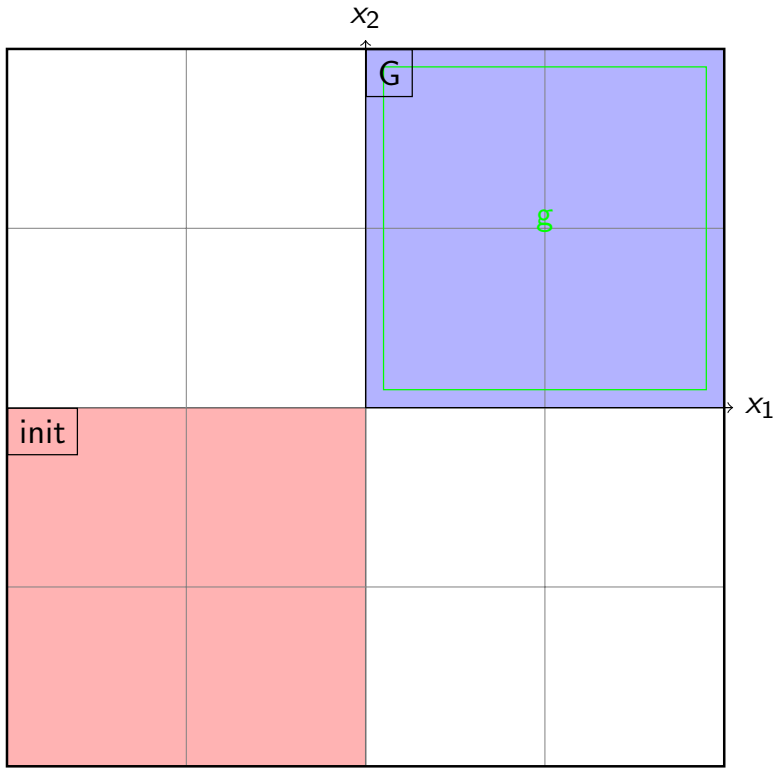
$\Rightarrow$ Strong cyclic planner with local fairness property

Escape time property in the cycle $\mathcal{C}$:
$\forall \{\mathbf{x}_k\}_k \in \mathcal{X}^*, \mathbf{x}_0 \in \mathcal{C} \Rightarrow \exists k \in \mathbb{N}, \mathbf{x}_k \notin \mathcal{C}$

Backward Reachability Algorithm - $\varphi = \Diamond\Box G$



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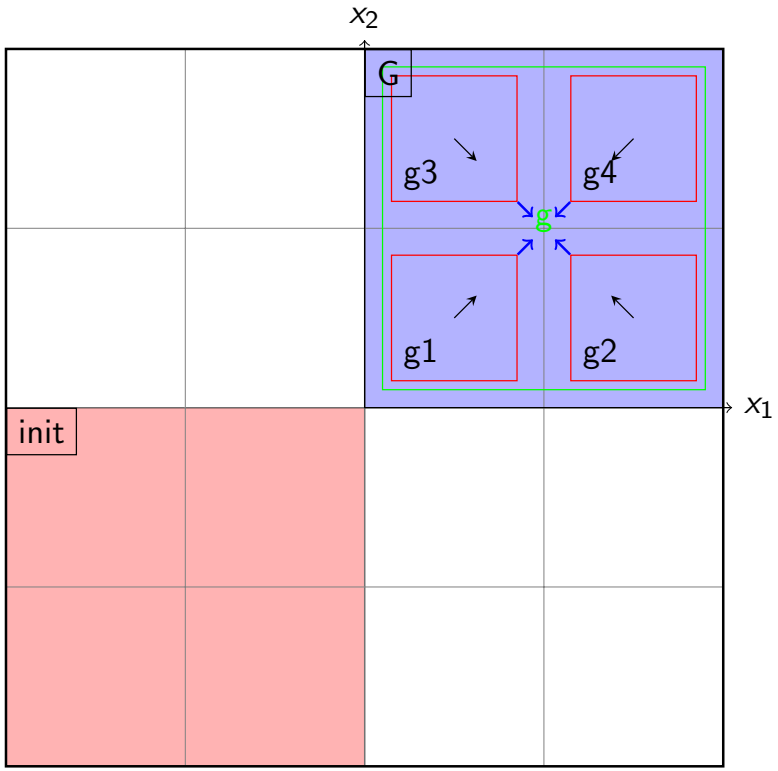

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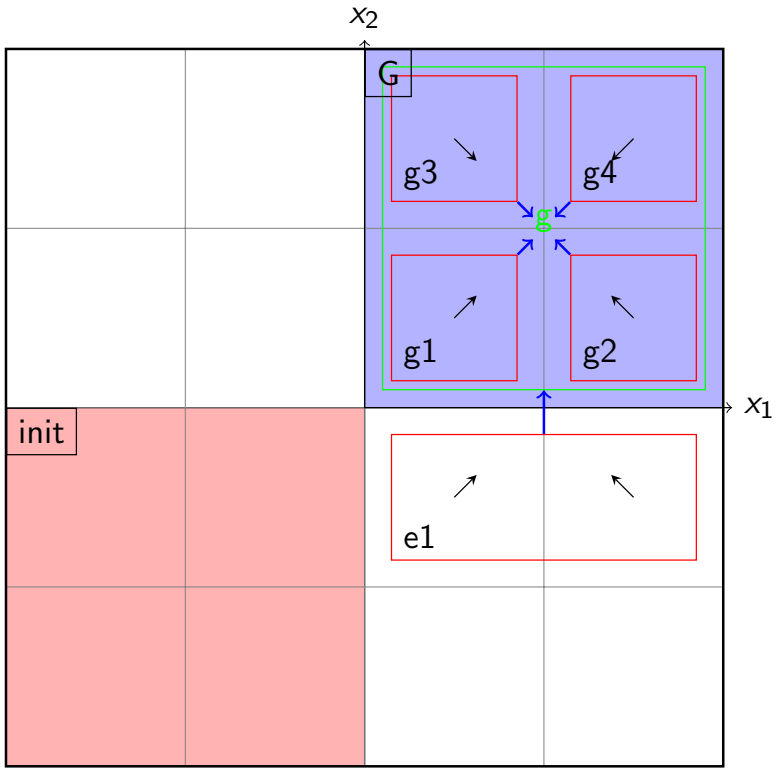

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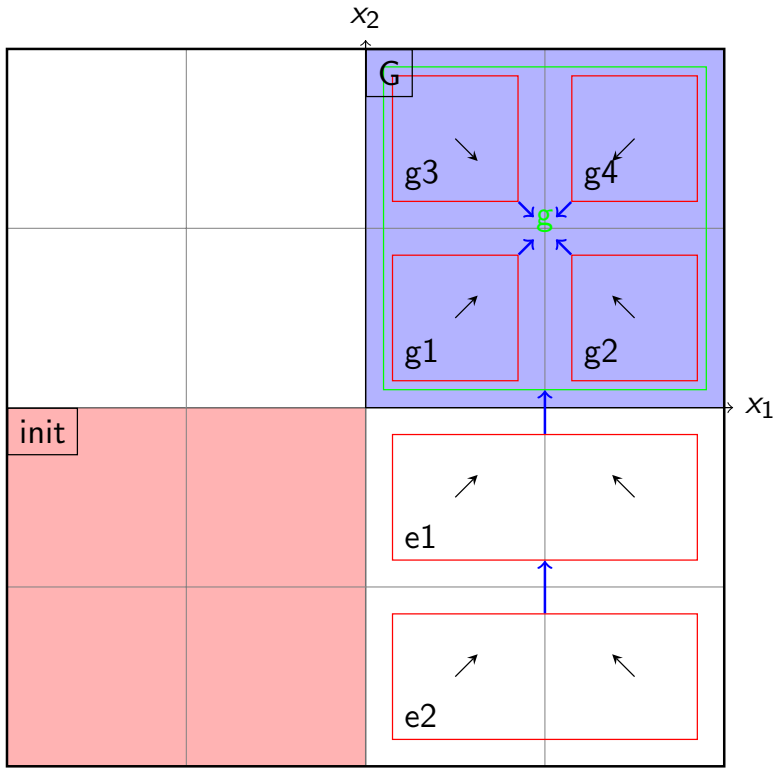

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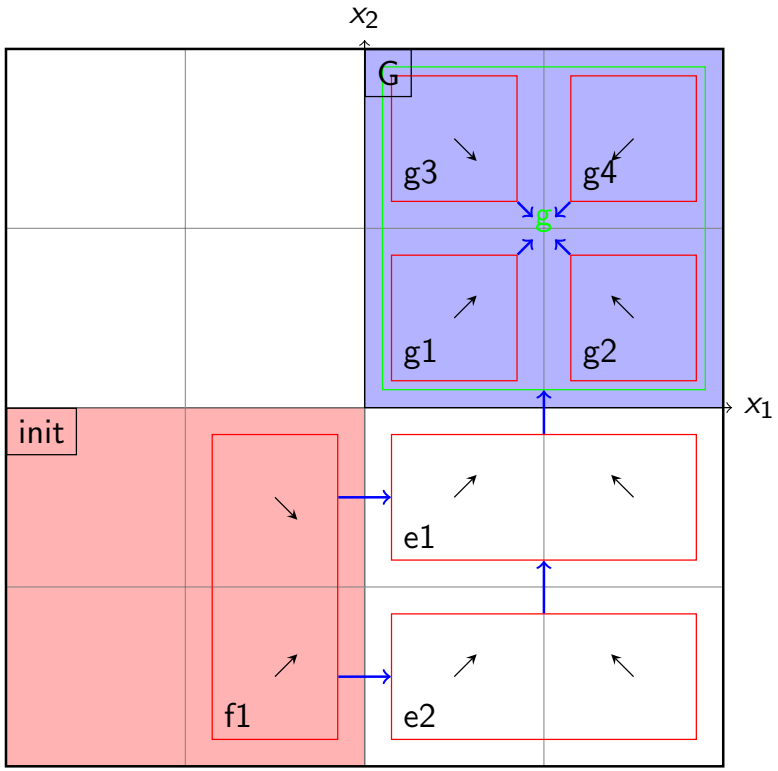

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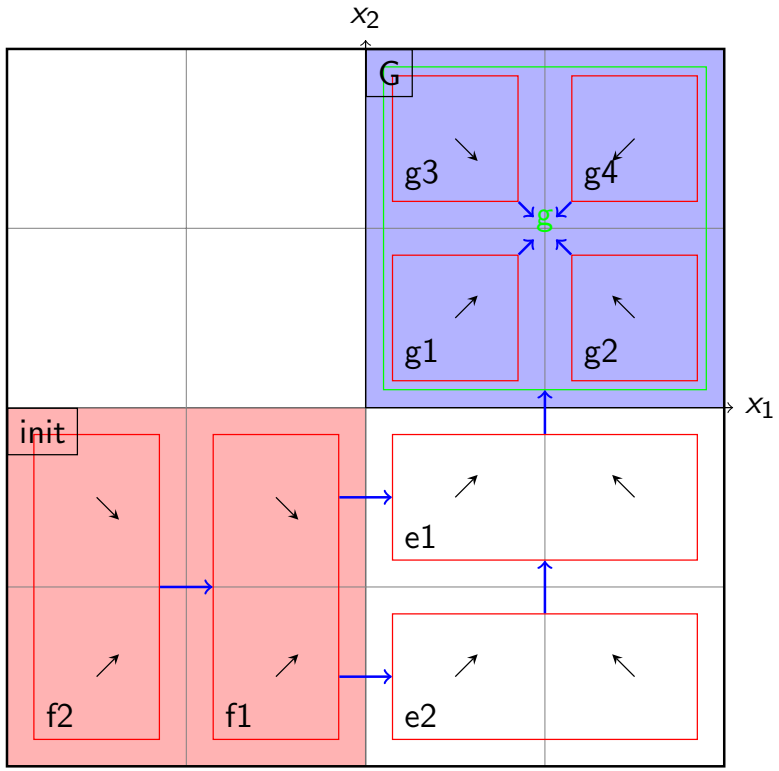
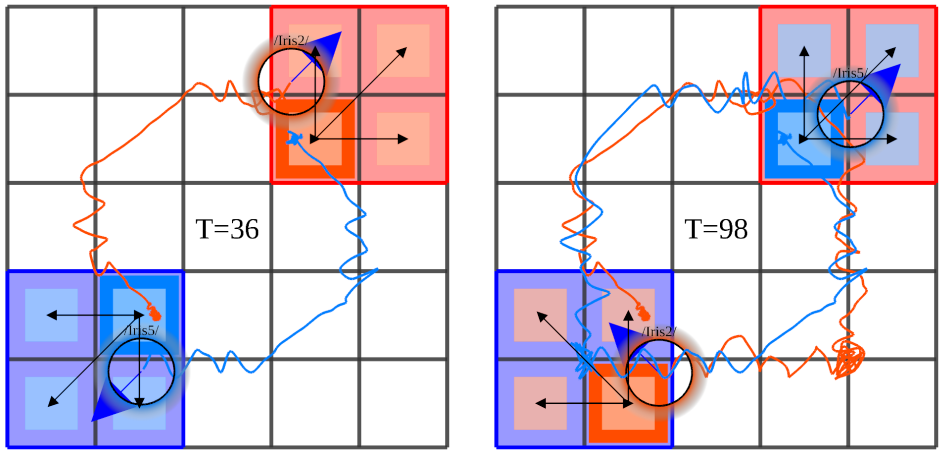


Multi agent experiment

- 0 memories (equivalent to the first integrator model)
- $\varphi = (\Box\neg out) \wedge (\Box\Diamond a) \wedge (\Box\Diamond b) \wedge (\Box\neg collide)$



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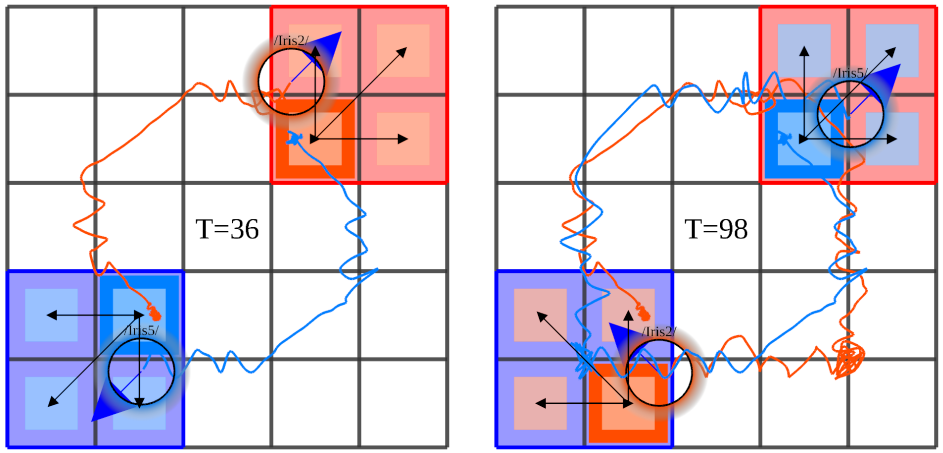

The video !

Tack så mycket!

Tack så mycket!
Du kan fråga vad du vill, jag ska svara (bara om det är möjligt).